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A risk and pay factor analysis of Washington State's Department of Transportation 1994 Standard Specification.

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A Risk and Pay Factor Analysis of Washington State's Department of Transportation 1994 Standard Specification

by

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Chapter 1

INTRODUCTION

In recent years, there has been a trend toward statistically based contract specifications in an effort to continually improve product quality, and provide additional value for the cost. The AASHO Road Test of 1958-1961 produced a sufficient number of unbiased test results of construction materials and the techniques used to install them to show for the first time their variability and relationship to the specifications. The result of these findings clearly demonstrated that the significance of certain items in the specification simply was not known, nor were the real standard or level of quality the specifications were supposed to guarantee [TRB, 1976, p 3]. This was the period in which the concept of performance based, or end-result specifications, was born and that a contract written with minimum standards would likely result in the same. The Blatnik Committee's discovery in 1962 that there was not 100 percent compliance with specifications almost lead to Congress passing a law making it a federal offense to knowingly incorporate nonspecification material in a highway project [TRB, 1976, p 3]. These were the genesis of today's developing sampling plans that estimate the true characteristics of materials and construction methods for which the specifications are written [TRB, 1976, p 3]. The only drawback to statistical sampling is that without a basic understanding of its characteristics and nuances, it can lead to undesirable consequences that may not be readily apparent to those designing and implementing the plan.

The research for this study put considerable emphasis on comparing Washington State's Department of Transportation Standard Specification with Military Standard 414. At the end of the project, it was discovered, unexpectedly, that the lack of association between the WSDOT specification and MIL-STD-414 was by design. In other words, the original intention of the specification writers was not to mirror exactly the sampling methods in MIL-STD-414 even though it appeared at first that it was. The primary reason for this was the recognition by the plan designers that the sample sizes that would likely be required using MIL-STD-414 simply were not economically feasible, therefore necessitating using small sample testing methodology. As will be demonstrated through these writings, the benefits of following MIL-STD-414 to the letter are lost, but the economic pay-back of smaller sample sizes compensate for that loss.

Statistical Sampling

This report was written to provide insight into using statistical sampling methods, their advantages and disadvantages, the pitfalls of equating expected pay to risk, as well as provide contractors an explanation of their responsibilities and the advantages to both contracting parties in a properly designed acceptance plan.

Statistical sampling plans are a tool by which a reasonable estimate of product quality can be made by measuring the characteristics of a randomly selected sample. Different sampling plans require different sample sizes for comparable levels of confidence in the results. It is here, the preliminary design stage, that a decision must be made to determine if it will be more expensive to make easy and quick measurements of a larger sample, or more meticulous measurements of a smaller sample. The results of the sample

measurements allow the inspector to make a decision, or a judgment sometimes called "sentencing" [Montgomery, 1991, p 551], about the body of material from which the sample was taken. Acceptance sampling is just what it says. It is not to be used to control the contractor's process capabilities. It is simply a means to decide if an owner should accept what the contractor is providing. The contractor can just as easily employ a statistical sampling technique to control the quality of the product before subjecting it to an owner's plan. These concepts will be discussed in more detail later in the report.

WA-RD 326.1

In 1989, the Washington State Department of Transportation (WSDOT) elected to implement quality assurance specifications on several asphalt paving projects. This was a test case in an effort to remove bias from inspection, and ensure a predictable level of quality. Positive feedback from both the contractors and state employees encouraged WSDOT to continue and broaden the use of statistically based specifications. The intent of WA-RD 326.1, "An Initial Evaluation of the WSDOT Quality Assurance Specifications for Asphalt Concrete" was to determine quantitatively any real changes in pavement quality as a result of the new specifications. The new specifications did indeed produce a modest improvement in quality based on six projects, three QA and three non-QA [Markey et al, 1994, p 1].

Quantifying Risk

One aspect of this report's research was to take WA-RD 326.1 one step further in an attempt to specifically quantify the statistical risks to both WSDOT, and the

contractors who operated under the new specification. In addition, the appropriateness of the pay factors for varying levels of product quality and sample sizes was also examined.

Insight for Developing a Sampling Plan

This report will be used to discuss how to develop a statistically based sampling plan. It will consider the costs of sampling and the relative impact of sample size, how to quantify what is acceptable or rejectable quality, and the best way to tie pay to quality level. The pitfalls of not properly applying established and defensible sampling methods will be identified and how to avoid them. Also a straightforward explanation of the concepts behind different sampling procedures will be given, and when it is appropriate to use or avoid them.

Chapter 2

BACKGROUND

The research for this project began with a literary search of all materials dealing with statistically based specifications relating to construction. It was soon discovered that the most relevant sources of information were Duncan, Montgomery, and MIL-STD-414. These were not necessarily construction oriented, but provided the background necessary for grasping the concepts inherent to statistical sampling. The sections that follow in this chapter provide the building blocks for understanding statistically based sampling.

OC Curves and How They are Developed

A properly designed acceptance plan, whether for variables sampling or attributes, can be represented by an operating characteristic (OC) curve. A variables sampling plan is one which tests and measures specific characteristics of the item sampled. It bases the decision to accept or reject on one characteristic at a time, from data which are computed such as mean, standard deviation, or percent defective. An attributes sampling plan is a go/no go approach. In this method, several characteristics may be measured, but the final result is simply acceptance or rejection for the sample item. Attributes procedures tend to involve things that are counted. Both sampling methods will be described in more detail later. The OC Curve represents how well the sampling plan discriminates against a defective product. In other words, given a specific sample size, and material with a certain quality level (percent defective), then the probability of accepting the material from which the sample was taken, at that quality level, can be read directly from the curve. There is

only one curve for each sample size. The entire curve demonstrates how the probability of acceptance changes with either an increase or decrease in product quality for that particular sampling plan. An OC Curve that has been constructed properly has the ability to account for any uncertainty associated with the fact that only a small portion of each lot is sampled [Weed, 1995, p 2]. What is meant by a properly designed acceptance plan, is simply one which was created following the guidelines in an accepted standard such as MIL-STD-414, "Sampling Procedures and Tables for Inspection by Variables for Percent Defective", or MIL-STD-105, "Sampling Procedures and Tables for Inspection by Attributes", or the principles of statistically based acceptance procedures outlined in most statistics books. There is enough flexibility built into these guidelines to be able to apply them in a wide variety of situations. If a sampling plan designer substantially departs from the guidelines indicated in the standards above, it may become impossible to accurately determine the plan's discriminatory power, and the risks assigned to the contractor and owner. Many plans tie the quality of a product, determined by the sampling plan, to how much the contractor will be paid. Generally, for a plan to work properly, there should be a bonus for exceptional quality, and substantially reduced pay at the level of quality that is just above the level where it would be rejected. The quality levels which determine these points will be discussed later.

α and β Risks

The concept of the amount of risk assigned to each party in a contract is described by the quantities α and β. These terms are also known as Type I and Type II errors, or more meaningfully, as seller's and buyer's risk respectively. An α error is one in which a true hypothesis is rejected, and a β error is one in which a false hypothesis is accepted [Mahoney, 1993, p 26]. In terms relating to contractors and owners, α risk is the chance that an owner might reject material from a contractor that should be accepted (seller's risk), and β risk is the chance that an owner will accept material that should be rejected (buyer's risk). Unlike normal, binomial, hypergeometric, or other type of distribution curve, an operating characteristic curve does not represent probability by area under the curve. Instead the chance, or probability of acceptance, is the distance from the curve down to the X axis, read from the Y axis. So the Y axis will always be a scale of the probability of acceptance from 0.00 to 1.00, and the X axis will represent the quality of the material either in terms of how much is "good", i.e. percent within limits, or how much is "bad", i.e. percent defective or fraction defective. Taking this a step further, the distance from the curve upwards to 1.00, is the probability of rejection. Sometimes the Y axis is represented by the term 1-α. Therefore for an OC Curve representing a specific sample size from a body of material, or lot, with a specific percent defective, can be used to determine exactly how likely it is that it will be accepted or rejected. Figure **2-1** below is an example of a typical OC Curve.

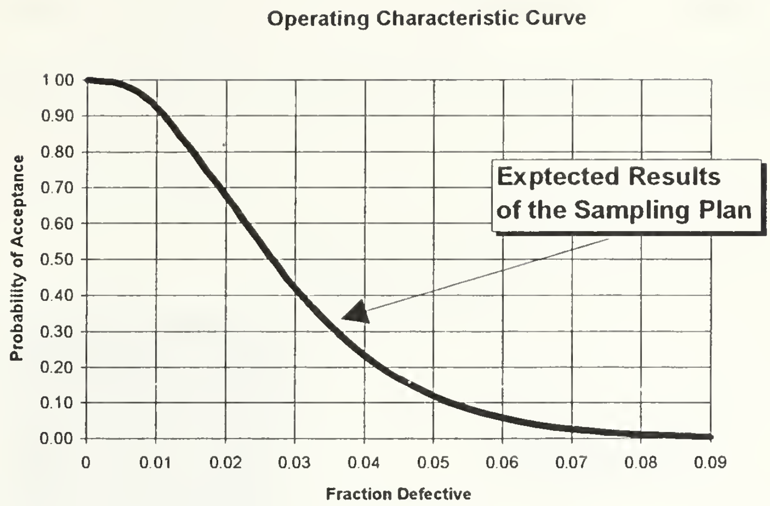

Figure 2-1 Typical OC Curve

This OC Curve says that at 0.5% defective (or 0.005 fraction defective), the probability of acceptance is 98.6%, at 2% defective the probability of acceptance is 67.7%, and at 7% defective the probability of acceptance is 2.6%. In an ideal sampling plan, a level of quality will have been established that is "acceptable". This will be at some point lower than perfect quality, because it is unreasonable to expect a contractor to be able to produce material completely free of defects. Recognizing this, an ideal OC Curve would accept material 100% of the time that is at or above the acceptable quality level. See Willenbrock Volume II for a more detailed discussion of the ideal OC Curve. The corresponding OC Curve would then reduce the probability of rejection to zero, or probability of acceptance to 100% for all material at or above the acceptable quality level. Likewise, the perfect sampling plan and OC Curve would reject everything below the acceptable quality level. Figure 2-2 below graphically demonstrates this concept where the acceptable quality level is 5% defective. This means that the contractor may provide

material that is up to, but no more than 5% defective with complete confidence that it will be accepted by the owner. Acceptable quality level, rejectable quality level, and zero defects will be discussed in more detail later.

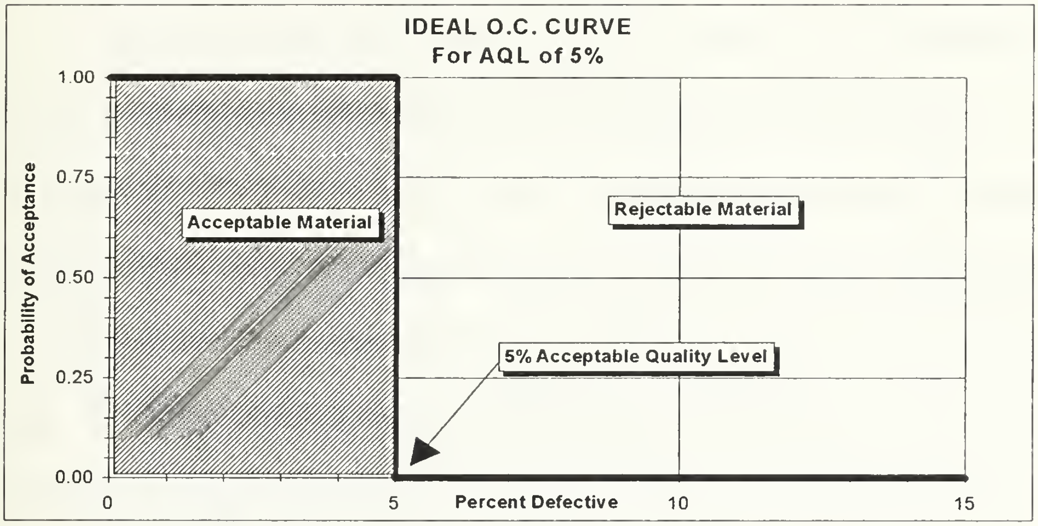

Figure 2-2 Ideal OC Curve

Identifying the Correct Statistical Model

Statistics describe different characteristics of naturally occurring data by first classifying them into a specific data distribution. Distribution curves may have different shapes, and will have different equations which describe their behavior. It is important that an acceptance plan designer understand enough about the process from which samples will be drawn so that the appropriate distribution is applied. Many times, simplifying assumptions are made that substitute one distribution for another, such as assuming that the data is normally distributed. As long as the plan designer understands when the disparity between "actual and assumed" are negligible can the substitution be

made with the confidence that it will not undermine the validity and enforceability of the plan. The following descriptions of binomial and hypergeometric distributions will help.

Binomial Distributions

The “normal” distribution, which is used more commonly than other distributions, represents continuous data. Discrete data is represented by the binomial and hypergeometric distributions, among others. Most field measurements are considered continuous limited only by the degree of precision of the instrument. These distributions are subsequently representative of the “pool” of data from which lots and samples are drawn. It is possible for a binomial and hypergeometric distribution to take on the exact same shape as a normal distribution, and in many cases is a close approximation. Normally distributed data is easier to manipulate, so making the assumption that the data is normally distributed is common. As has been the experience of those involved in construction, the vast majority of construction characteristics are in fact normally distributed, so this simplifying assumption is not a stretch of reality. Typically field data tends to not be normally distributed only when there is some sort of physical limitation such as zero percent air voids, or minimum cover over reinforcing steel. An example of discrete data and continuous data is included later.

Normally distributed data comes from a universe that is infinite in size. A binomial distribution is the probability distribution for a continuous, or theoretically infinite process operating randomly over time. The random operation can be visualized as one which produces some product where on average, say, 5% are defective. So if you were to draw lots from this process, each lot would on average have 5% defective. This is how a

consumer would view the operating characteristics of his sampling plan when buying a steady stream of material from a supplier [Duncan, 1986, pp 164, 165].

Type B OC Curves

There are two categories of operating characteristic curves, Type A and B. Type B curves are built from probability of lot acceptance based on the binomial distribution. The formula for the binomial distribution is

$$P\left(\frac{X}{n}\right) = \frac{n!}{X!(n-X)!} p'^{X} (1-p')^{n-X} \qquad \textbf{Equation 2-1}$$

where P(X/n) = probability of X nonconforming in a sample of n items
X = number of items nonconforming in the sample
n = sample size
p' = lot fraction defective
[Duncan, 1986, pp 90-91]

For instance, if the sample size is 10, and it is known that the lot has 5% of its items defective, and 3 of the 10 items sampled were found defective, Equation 2-1 would give the probability of finding those 3 defective items. As will be demonstrated later, it is the summation of probabilities from zero defective, up to the designer's tolerance, that yields the probability of acceptance.

Hypergeometric Distributions

Unlike the binomial distribution, the hypergeometric distribution is much more limited in scope. The data it represents is assumed to have been drawn from a pool that is limited, or finite in size, and that the samples drawn from it are not replaced. This would be situations such as a one time product run, or an item that is manufactured between

changes affecting production. This is also how a consumer would view the operating characteristics of a sampling plan when isolated lots of material are purchased, or when the consumer thinks about the quality of individual lots [Duncan, 1986, p 165]. In this case, it might be appropriate to assume that the material produced by one job mix formula, JMF, from WSDOT's specification could be described by the hypergeometric distribution. In reality though, WSDOT uses about 80 pounds of material for each test from a lot which may be thousands of tons. For all practical purposes this could safely be approximated by the normal distribution.

Type A OC Curves

Type A operating characteristic curves are based on the hypergeometric distribution. The formula for the hypergeometric distribution is

$$P\left(\frac{X}{n}\right) = \frac{C_{n-X}^{N-m} C_X^m}{C_n^N} = \frac{\dfrac{(N-m)!}{(n-X)!(N-m-n+X)!} \bullet \dfrac{m!}{X!(m-X)!}}{\dfrac{N!}{n!(N-n)!}} \qquad \textbf{Equation 2-2}$$

where P(X/n) = probability of X nonconforming in a sample of n items
X = number of items nonconforming in the sample
N = lot size
n = sample size
m = lot fraction defective
C_X^m = number of combinations of X out of m
[Duncan, 1986, p 94]

Because this formula is more difficult to manipulate, and that some calculators and spreadsheets are limited by the size factorial (e.g. 5! = 5*4*3*2*1=120) it can handle, it is

desirable not to work with the hypergeometric distribution, especially since it was discovered that Microsoft's' Excel 5.0 was only capable of working with factorials up to 170!. As will be shown later, the Type B OC Curve is still a good approximation in many circumstances.

Effects of Large Sample Sizes on Type A OC Curves

Previously it was shown that the hypergeometric distribution and binomial distribution are the basis for the probabilities of acceptance of Types A and B curves. For the most part, Type B curves are almost exclusively used in statistically based acceptance plans. The reason for this is that as the lot size increases, the lot has a diminishing impact on the behavior of the OC Curve. In fact the general "rule-of-thumb" is that if the lot size is at least ten times the size of the sample, the Type A and B curves are indistinguishable. The Type A curve will always be below the Type B curve, or rather, the probability of acceptance will always be lower for a Type A curve than for Type B. But as mentioned above, the difference is only significant if the lot size is small relative to the sample. Figure 2-3 below is used to demonstrate this difference, where N is the lot size, n is the sample size, and c is the acceptance number [Montgomery, 1991, pp 562-563]. The acceptance number is the maximum number of defective items tolerable in one sample.

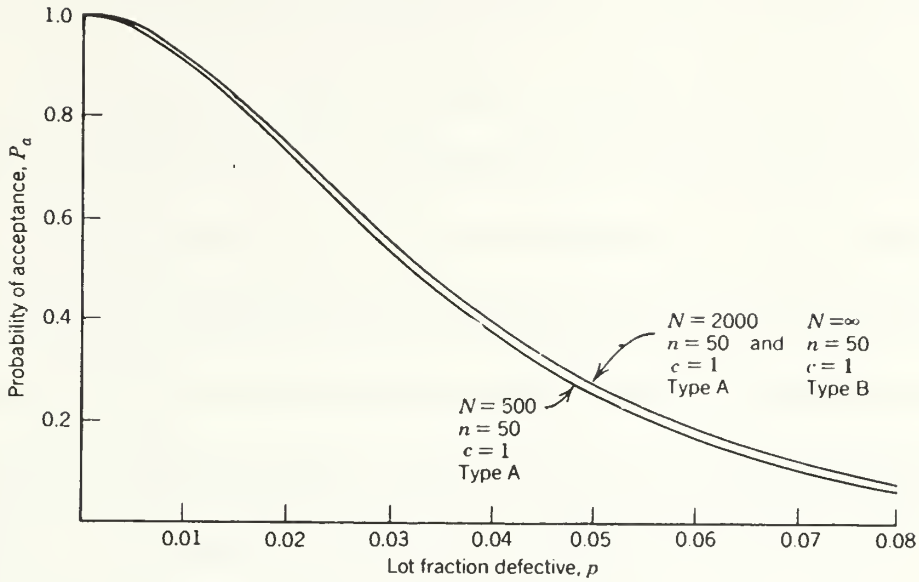

Figure 2-3 Type A vs Type B OC Curves [Montgomery, 1991, p 563]

And finally to bring these different types of curves into perspective on how they are used; Type A curves are typically not used because the hypergeometric distribution is difficult to work with. Besides, most of the time a Type B curve will suffice because of the relative differences in size between the samples and the lots from which they came. So generally, Type B curves are used in both Type A and B situations.

Discrete/Continuous Data

As important as it is that a sampling plan designer understand the type of distribution which applies, is understanding whether the situation deals with discrete or continuous data. As mentioned earlier, hypergeometric and binomial distributions represent discrete data, while the normal distribution, represents continuous data. Discrete data are data that can assume only an integer value. Continuous data can assume a value between two limits, limited only by the precision of the instrument [Blank, 1980, p

8]. An example of discrete data would be the number of marbles in a bucket, and an example of continuous data would be the number of minutes it takes to run a mile. The reason these distinctions are mentioned is because most acceptance plans make the simplifying assumption that the data is normally distributed (i.e. continuous data), when in reality it may not be. The primary reason this assumption is made, is because the normal distribution is by far the easiest to manipulate, and for which probability tables are readily available. Plus it is also reasonable to expect to find that a binomial distribution has been substituted for a hypergeometric distribution (as a close approximation), and then that a normal distribution has been substituted for a binomial distribution, also as a close approximation. If these two successive substitutions are made, this results in a distribution that represents continuous data from an infinite universe being assumed as equivalent to a set of data that may be discrete and from a finite universe. It is only when a sampling plan designer recognizes these difficulties, that the appropriate model can be applied, or at least that assumptions can made that will not significantly affect the integrity of the plan.

AQL/RQL

As different organizations began developing statistical specifications, they quickly discovered that it was very difficult to define a single level of quality that clearly distinguished between acceptable and rejectable work. Instead it was much easier to define a range of quality where at the high end it was called an acceptable quality level, AQL, and at the low end, below which the quality was poor enough to reject it, the rejectable quality level, RQL. In between these two levels of quality, the work was considered to be poor enough to justify a pay reduction, but not so poor as to warrant

rejection or replacement [Weed, 1994, p 1]. This was the genesis of the concept of adjusted pay which provided a means to accept slightly defective work or material, for a reduced pay amount which was agreed upon in the contract documents.

The next question then becomes, what are the appropriate sizes of buyer's and seller's risks? There are no hard and fast rules, but generally as a means to determine appropriate levels, the defects in question must first be classified. The following distinctions are made in "Statistically Oriented End-Result Specifications", TRB, 1976:

> Critical: This defect will make the product dangerous to use
> Major: This defect will seriously impair performance of the item
> Minor: This defect may impair performance but not seriously
> Contractual: This defect is likely to have insignificant effect on performance

MIL-STD-414 describes defects as follows

> A defect is a deviation of the unit of product from requirements of the specifications, drawings, purchase descriptions, and any changes thereto in the contract or order. Defects normally belong to one of the following classes, however defects may be placed in other classes:
>
> Critical Defects. A critical defect is one that judgment and experience indicate could result in hazardous or unsafe conditions for individuals using or maintaining the product: or, for major end items units of product, such as ships, aircraft, or tanks, a defect that could prevent performance of their tactical function.
>
> Major Defects. A major defect is a defect other than critical, that could result in failure, or materially reduce the usability of the unit of product for its intended purpose.
>
> Minor Defect. A minor defect is one that does not materially reduce the usability of the unit of product for its intended purpose, or is a departure from established standards having no significant bearing on the effective use or operation of the unit.
>
> [MIL-STD-414, 1957, p 1]

Recognizing that a critical defect should have a much lower acceptable quality level than a minor one, assuming percent defective, MIL-STD-414 provides plans, and the OC

Curves which describe them, for AQL's of 0.04%-15.0% which means 0.04%-15.0% of the characteristics tested can be defective, but still considered acceptable depending on the criticality of the characteristic. This is one reason why a variables inspection method, such as MIL-STD-414, requires a separate plan for each quality characteristic in question. Variables sampling plans will be discussed in more detail later, but generally, variables are quality characteristics that can be measured on a numerical scale, and attributes are quality characteristics that are expressed on a "go, no-go" basis [Montgomery, 1991, p 553].

Some sampling plans do not make distinctions between the varying levels of defects, and broadly assign a very typical value of 5% risk at the AQL to the contractor. Presumably this represents the risk that the plan designer wishes for all the quality characteristics being measured, which may not be appropriate. The problem is that for varying sample sizes it is very difficult to maintain control of the discriminating power of the sampling plan unless two points on the OC Curve are predetermined.

Why a Zero AQL is not Practical

As desirable as it may sound to have an acceptance procedure requiring 0% defective, in reality unless it represents a quality characteristic that could determine a life or death situation, it is not practical. In theory, the ideal OC Curve could be reached provided there is 100% error free inspection. It is clear that this level of inspection will be much more expensive than random sampling, and that all processes have some inherent variability making error free inspection unlikely. Figure 2-4 demonstrates the effect zero tolerance has on the shape of an OC Curve.

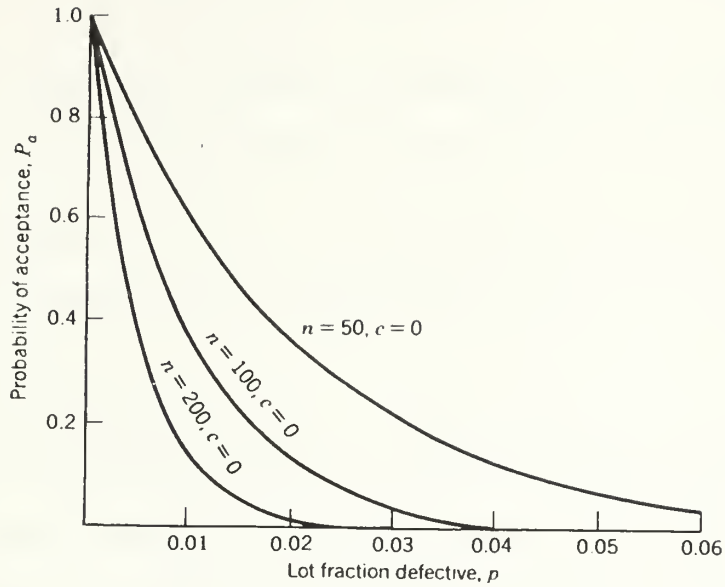

Figure 2-4 Effect of Zero Tolerance on OC Curves [Montgomery, 1991, p 564]

Generally, sampling plans that have zero tolerance will be convex through their range. It can be readily seen that the probability of acceptance rapidly decreases for relatively small percents defective. This can be a severe consequence to the contractor and should be expected to be reflected in contractor bids [Montgomery, 1991, p 563].

Using AQL/RQL in Developing OC Curves

As noted earlier, the OC Curve's function is to demonstrate graphically the probability of accepting a product that is provided at a certain level of quality. OC Curves are generally designed so as to pass through, or very near two points that are important to the plan designer. The points which are easiest to quantify are those at the rejectable and acceptable quality levels where the plan designer wishes to assign specific risk based on the criticality of the characteristic. Theoretically any two points could be used, but usually

the designer begins with the desired α and β risks at the AQL and RQL. Preferably a sufficient study of the characteristic to be measured should be conducted to ascertain what quality levels are appropriate. This means that the AQL and RQL should be realistic, as should the specification limits, and not just perpetuate limits used in previous acceptance procedures. If historical data is available, the FHA uses as a rule of thumb the deviation of the mean from the specification plus two standard deviations as specification limits. An example of perpetuating limits which are unnecessary might be a specification which requires the use of a high quality, expensive aggregate for a secondary road which could realistically be constructed with a local, cheaper aggregate with satisfactory results. Provided there is no data to support choosing a specific level, typically α is 5% and β is set at a minimum distance of 2σ from the mean [TRB, 1976]. According to Willenbrock, for non-critical products, α and β are usually chosen as 0.5% and 10% respectively [Willenbrock, 1976, p20.33]. For a non-critical quality characteristic it is unusual for α to be 0.5%, so perhaps that author meant 5% instead. If not, this demonstrates the variability in references for choosing buyer's and seller's risks.

The contractor will always be concerned with the level of quality, or quantity of material allowed to be defective and still have a predetermined chance of having that material accepted. Or in other words, at the 95% probability of acceptance, the contractor might be interested in the corresponding percentage of defective material since this is typically where the AQL is set. It should be noted that the AQL is **NOT** a property of the acceptance plan. It is rather the lowest level of quality that the owner or buyer will accept as a process average. Also the AQL is **NOT** intended to be a specification or target value

for the contractor. It is instead simply a standard chosen by the owner to sentence the material being presented for inspection. OC Curves are designed so there is a high probability of acceptance at the AQL, and a low probability of acceptance at the RQL. Again, the RQL is not a characteristic of the sampling plan, but a standard by which the owner will judge poor material offered for inspection [Montgomery, 1991, pp 561-562].

Designing a Specified OC Curve

Once a plan designer has determined the appropriate levels of risk for a certain quality characteristic at the acceptable and rejectable quality levels, those points can be used to design an OC Curve that passes through or close to them. Designing an OC Curve is the same thing as designing a sampling plan. For attributes sampling, given a sample size, and an acceptance number, it is possible to calculate the varying probabilities of acceptance using the binomial equation, Equation 2-1. For example, given a sample size n=89, and an acceptance number (the maximum number of defective items tolerable in a sample) c=2, then the probability of acceptance is the probability that d is less than or equal to c or

$$P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

and for a lot fraction defective where p = 0.01, n = 89, and c = 2 then

$$P_a = P\{d \le 2\} = \sum_{d=0}^{2} \frac{89!}{d!(89-d)!}(0.01)^d (0.99)^{89-d}$$

$$= \frac{89!}{0!89!}(0.01)^0(0.99)^{89} + \frac{89!}{1!88!}(0.01)^1(0.99)^{88} + \frac{89!}{2!87!}(0.01)^2(0.99)^{87}$$

$$= 0.9397$$

[Montgomery, 1991, p 559]

In other words, the probability of accepting a lot that is 0.01 fraction defective with a sample size of 89 and able to tolerate 2 defective in the sample, is the summation of the probabilities of 0 defective, 1 defective, and 2 defective.

Table 2-1 below shows computed probabilities for fraction defective from 0.005 to 0.090 for n=89 and c=2.

Table 2-1 Probabilities of Acceptance [Montgomery, 1991, p 559]

Fraction Defective, p	Probability of Acceptance, P_a
0.005	0.9897
0.010	0.9397
0.020	0.7366
0.030	0.4985
0.040	0.3042
0.050	0.1721
0.060	0.0919
0.070	0.0468
0.080	0.0230
0.090	0.0109

Sample as a Fixed Percentage of Lot Size

Another potential problem besides setting AQL at zero is establishing a sample size as a fixed percentage of the lot size. The problem with this is traced back to how an OC Curve behaves with varying sample sizes. It was stated previously that an OC Curve becomes more discriminating, or rather the slope steepens, with a larger sample size. In effect then, the level of protection afforded both the contractor and owner will vary depending on sample size [Montgomery, 1991, pp 564-565]. This is illustrated in Figure 2-5. In this figure the lot sizes vary from 100 to 1000, and for each the sample is fixed at 10% of the lot size with c=0, or zero AQL.

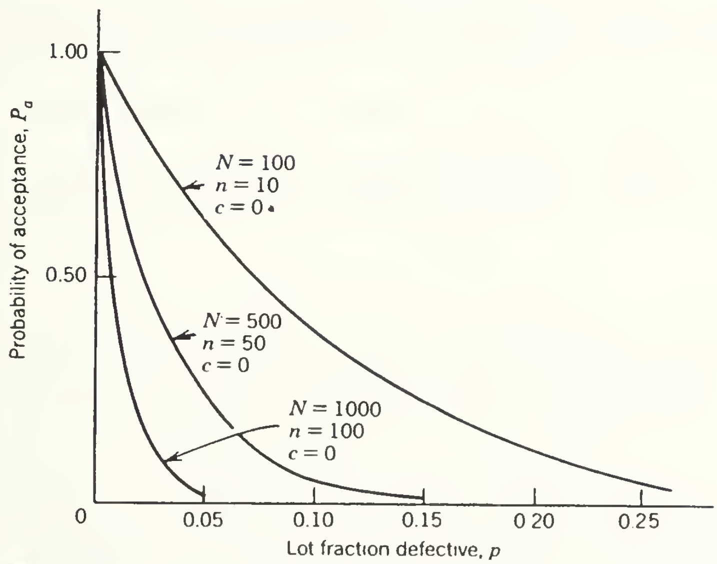

Figure 2-5 Sample as a Percentage of Lot Size [Montgomery, 1991, p 564]

The resulting curve is more discriminating, or steeper for larger sample sizes, so although the intent may have been to simplify the sampling plan, the effect is a drastically changing level of protection for the contractor at small fractions defective which may not have been

intended. It should also be noted though, that the larger sample size gives a better "picture" of lot quality.

Single and Double Specification Limit Plans

It is important that a few additional sampling concepts and terminology be described. A sampling plan will fall into one of two categories; either a single specification limit, or a double specification limit plan. A single specification limit plan is one where the quality characteristic is compared to a single limiting value. In this case acceptance is based on whether the sample quality characteristic should be less than or equal, or greater than or equal to the specified value. For example, in an asphalt concrete pavement specification, the compaction requirement will be specified as greater than or equal to some minimum value. A manufacturer that produces plastic soda bottles might have a specification which has a minimum psi rating. These are both single specification limit plans.

A double specification limit plan is used when the quality characteristic must fall within a range. The range is specified by a lower and upper limit. An asphalt concrete pavement specification again affords a good example where characteristics such as asphalt content, or gradation is specified as being acceptable as long as it falls between two limits.

Single and double specification limit plans are not to be confused with single or double sampling. They sound similar, but have entirely different meanings. Single and double sampling will be discussed later.

Relationship to k and M Sampling Methods

There are two methods used in statistical sampling plans. They are described as Form 1 and Form 2, Procedure 1 and Procedure 2, and as the k and M methods. The three designations are essentially identical, but are referenced by these different names depending on the publication. Here they will be referred to as the k and M method since that is how they are described in MIL-STD-414. The k method is essentially a distance test, and the M method is an area test. Using MIL-STD-414 procedures, a minimum distance, k, from the mean of the sample data to the lower specification limit (or upper specification limit) is obtained. If the sample data indicates that the distance from the sample mean is greater than k, the lot should be accepted. This concept is illustrated in Figure 2-6.

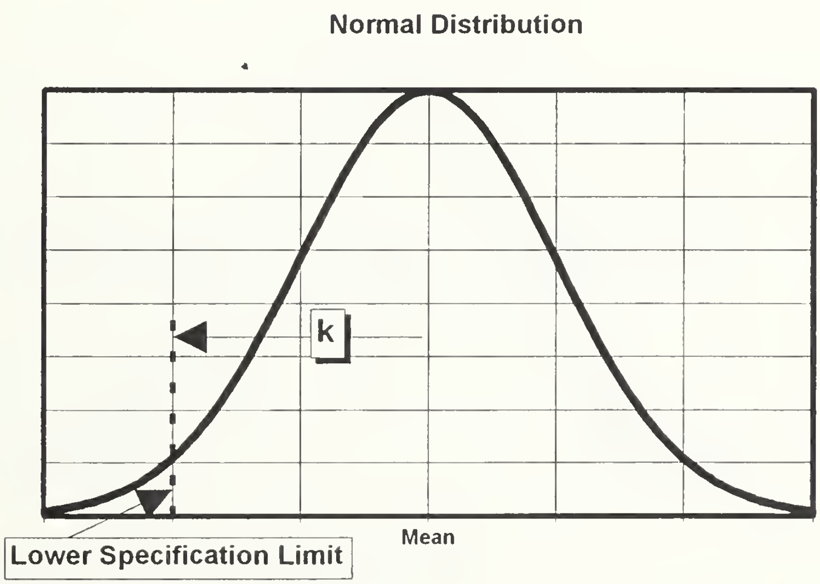

Figure 2-6 Single (Lower) Specification Limit for a Normal Distribution

Since the total area under the curve represents 100% of the sample, it follows that the further the mean is from the specification limit, the less area, or less out-of-specification material, will be under the curve at the tail beyond the specification limit. The same is true for an upper specification limit.

On the other hand, the M method uses a maximum area under the tail(s) of the distribution marked by the upper and lower specification limits. In this method, MIL-STD-414 gives a maximum area (represented as a percentage) not to be exceeded. By use of Figure 2-7 it can be seen that the shaded areas, together representing the maximum area not to be exceeded, M, can be achieved even if the mean of the distribution shifts slightly left and right. That is because this sampling procedure does not give a maximum or minimum value for the upper and lower limit tail areas, only a total area.

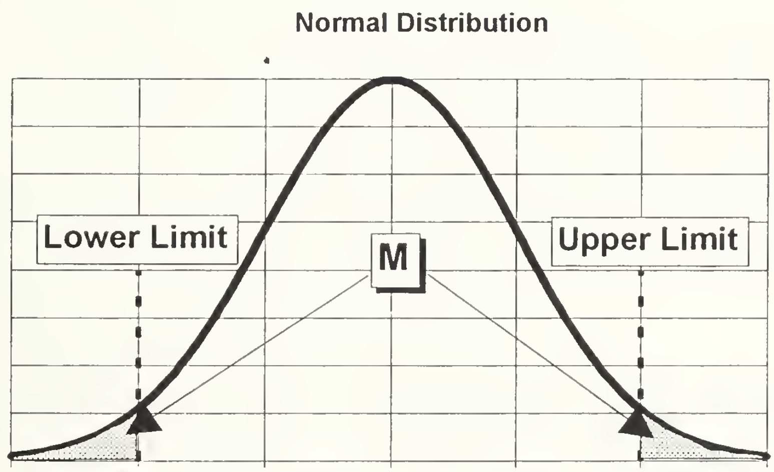

Figure 2-7 Double Specification Limit

So a shift in the mean to the left will increase the amount of material falling outside the lower specification limit, and lower the amount falling outside the upper

specification limit. As long as the total area under the tails does not exceed M, the lot should be accepted. Further, Figure 2-6 and Figure 2-7 graphically demonstrate the concept of single and double specification limit plans.

Single Sampling vs Other Methods

Single sampling is one of several available in statistical acceptance procedures. A single sample does not imply a sample of one unit. A single sample could be one unit, or thousands, and is usually referred to by the small letter "n". An acceptance plan based on single sampling relies entirely on the integrity of the data obtained by observing the characteristics of that one sample. Single sampling is adequate for most situations.

By contrast, double sampling is a procedure by which a second sample *may* be required before the lot can be sentenced. If it is found that the sample has more defective than that which would allow an unquestioned "pass" (similar to AQL), but less than that which would require outright rejection (similar to RQL), a second sample would be taken to determine if the combined percents defective from both samples was above or below the rejection limit [Montgomery, 1991, p 571].

A multiple sampling plan is an extension of the double sampling plan. This plan might involve more than two samples. If at any stage of the sampling the percent defective is less than the acceptance number (the maximum number of defectives tolerable in a sample), the lot is accepted. If at any stage the sample equals or exceeds the acceptance number, the next sample is taken. This procedure requires that a limit be placed on the maximum number of samples that may be taken [Montgomery, 1991, p578].

Sequential sampling is an extension of both double and multiple sampling. This method requires a sequence of samples to be taken from the lot, the number of which is determined entirely by the results of the sampling process. Theoretically, this procedure could perpetuate itself until the entire lot was sampled [Montgomery, 1991, p 579].

The description of the three previous sampling procedures gives an indication of what is available, but by no means describes all acceptance procedures. Obviously the more sampling that is done, the more expensive it will be. The plan designer must make a decision early on to determine the cost trade off of multi-tiered sampling over the expected increase in confidence in the sampling results. Should the plan designer wish to pursue one of these alternate methods, it should be recognized that MIL-STD-414, inspection by variables, only offers single sampling procedures. MIL-STD-105, inspection by attributes, offers single, double, and multiple sampling.

Chapter 3

QUANTIFYING RISK

The focus of this research began by investigating the statistical risks, α and β, assigned to the asphalt paving contractors and the Washington State Department of Transportation, (WSDOT). Although many agencies have converted to QA specifications, there is a perception that a lack of understanding exists about the risks. The original goal was to build on the work contained in WA-RD 326.1 and "back out" the risks using data and quality/pay indexes in the report appendices. After concluding the research, however, it was discovered that the pay factor tables had been designed based on small sample theory, or t-distribution, which considers the skewing effects of relatively small samples.

Secondary to determining risk, this paper is intended to be used, at least in part, as a tool to describe acceptance plan risks in a way which will be easily understood by anyone with a technical background, but not necessarily versed in statistics. As such it will enable those in public agencies charged with designing and developing statistical acceptance plans to develop an awareness of some of those aspects which demand attention, such as potential pitfalls associated with an unclear understanding of operating characteristic curves. This section will examine WA-RD 326.1 for a determination of WSDOT's associated α and β risks.

Normal/Hypergeometric/Binomial/t Distributions

Although WA-RD 326.1 specifically states that WSDOT's asphalt concrete specifications are based on MIL-STD-414 with modifications, it was still studied to determine to what degree this was true. MIL-STD-414 does not include, and understandably so, a discussion of hypergeometric, binomial, t, and normal distributions. It was written for those seeking an alternative to traditional non QA inspection methods, and relied on the agency using the standard to provide the expertise needed to determine when it was appropriate to apply. In other words, someone had to know whether the data collected by random sample was produced by a process which closely approximated a normal distribution. And if it did not, either use an alternative sampling method, or recognize that the results could not accurately be quantified by the OC Curves included in the standard. Although Duncan states that MIL-STD-414 can still be used in non-normal situations, the further the departure from normal, the less confidence there is in the OC Curves which describe the acceptance plan's behavior [Duncan, 1986, p 256]. Chapter 2 described the relationship between normal sampling data, whether from a continuous process or single lot, and the resulting distribution; binomial or hypergeometric. WSDOT's 1994 specification states that:

> For the purpose of acceptance sampling and testing, a lot is defined as the total quantity of material or work produced for each job mix formula (JMF), placed and represented by randomly selected samples tested for acceptance [Standard, 1994, 5-04.3(8)A, p 5-22].

This potentially places the lot from which data is obtained by WSDOT's sampling method in the hypergeometric category since the material produced for one JMF is

a lot of finite size, and the sample material is not replaced. But despite this, at least as far as testing is concerned and the relative differences in sizes of the sample and lot, the nature and method of data collection correctly creates the presumption of a normal distribution.

Type A/Type B OC Curves

In many engineering circumstances, and applied statistics, there are simplifying assumptions made to make design and analysis manageable. That is the case with MIL-STD-414, and MIL-STD-105. Earlier it was demonstrated that Type A OC Curves represented sampling plans with data resulting from a finite universe, or hypergeometric distribution, and Type B OC Curves represented data from a continuous process, with sample data resulting from the binomial distribution. It was also demonstrated earlier that as the lot size increases, the acceptance plan approaches the Type B OC Curve. Only Type B OC Curves are found in both Military Standards. This is a safe approximation, that is to use Type B Curves, as long as the lot is at least ten times the sample size, and the sample is not small [Montgomery, 1991, p 562].

Nomographs

It is possible to design an acceptance sampling plan with a specified OC Curve. Since the OC Curve simply represents the probability of acceptance over a range of quality from perfect to poor, then either the binomial or hypergeometric summation formulas, Equation 2-1 or Equation 2-2, are used. Only using the formulas is less than simple. They are tedious, time consuming, and must be repeated many times over since each OC Curve

represents only one sample size and one acceptance number for a desired α and β. Plus since the equations are nonlinear, there is no direct solution [Montgomery, 1991, p 565]. There is a simpler, though less accurate way which is to use either the binomial or hypergeometric nomographs. Figure 3-1 is used for attributes sampling plans, and Figure 3-2 is used for variables.

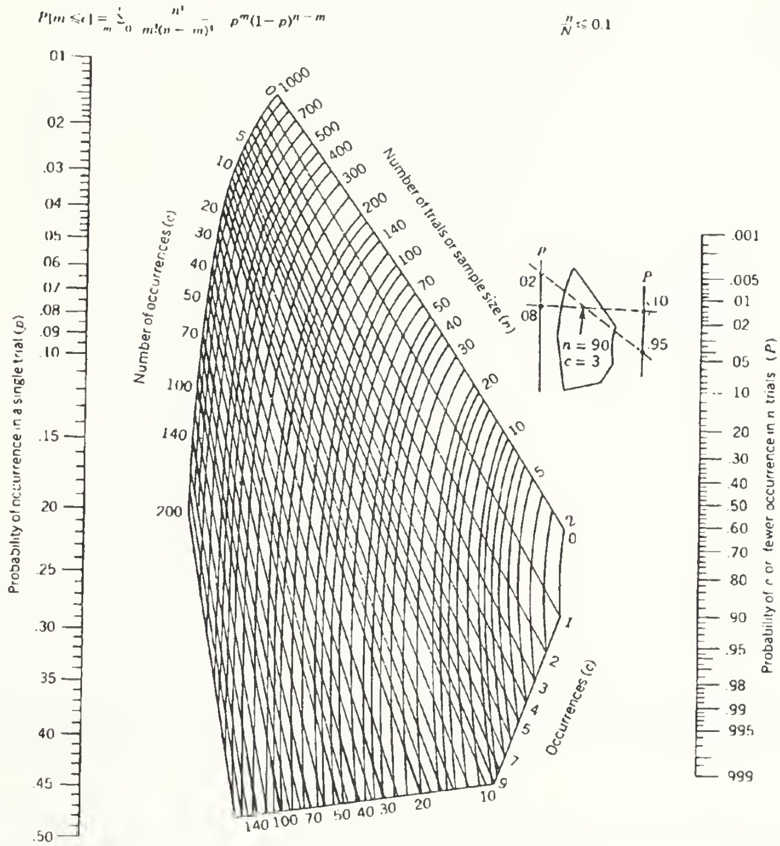

Figure 3-1 Binomial Nomograph-For Attributes Sampling Plans

[Montgomery, 1991, p 566]

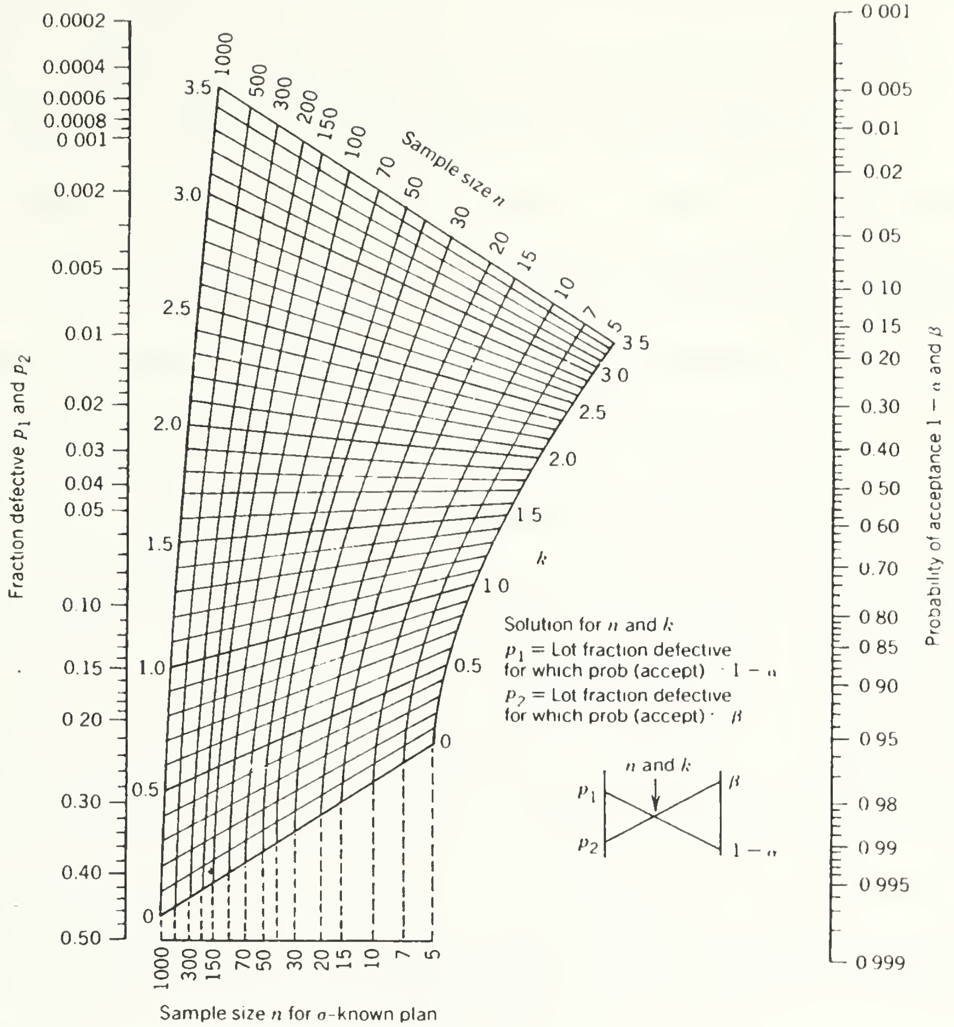

Figure 3-2 Hypergeometric Nomograph-For Variables Sampling Plans [Montgomery, 1991, p 627]

To use the attributes nomograph, Figure 3-1, first a line is drawn from the fraction defective desired for the AQL on the left scale, to the desired probability of acceptance (1-α) at that fraction defective on the right scale, then another line from the fraction defective desired for the RQL on the left scale, to the desired probability of acceptance (β) at that fraction defective on the right scale. Note that for the AQL the probability of acceptance

is 1 minus the seller's risk. Then trace from the intersection of these two lines curving up and to the right to read the required sample size, and then again from the intersection down to the right or up to the left to read the acceptance number (recall that the acceptance number is the maximum defectives tolerable in a sample). It is apparent that the intersection will not always land cleanly on the lines in the nomograph. That means there are several sampling plans available that will closely approximate the desired results [Montgomery, 1991, pp 565-566].

Figure 3-2 *by itself*, works only for sampling plans using the k method. The procedures for using this nomograph are exactly the same as for Figure 3-1, only instead of reading an acceptance number, it gives a minimum value for k. Note that two values for the sample size, n, can be read from this nomograph. Reading down, n is given for situations where the process standard deviation (σ) is known, and reading up, for when it is not known [Montgomery, 1991, pp 626-628]. When reading for unknown standard deviation, trace upward from the intersection following the curved lines of the nomograph. If the standard deviation is known, the sample size is read directly, and vertically, below the intersection; do not follow the curved lines of the nomograph or the results will be the same as if reading up. As might be expected, when the standard deviation, σ, is not known, there is greater uncertainty which requires a larger sample size for the same level of confidence. After the sample is taken, the mean and standard deviation are calculated, and are then used to determine Z. Or rather Z is calculated using Equation 3-1, in this case where there is a single lower specification limit.

$$Z_{LSL} = \frac{\bar{x} - LSL}{\sigma}$$ **Equation 3-1**

where: Z is a standard normal deviate
$\bar{x}$ = sample mean
LSL = lower specification limit
σ = sample standard deviation

If Z is $\geq$ k, then the lot is accepted.

It is important to note that for either nomograph, given a sampling plan, meaning a sample size n, and either acceptance number, c, or minimum k, the probability of acceptance for any fraction defective can be read directly from the nomograph.

k vs M Method

Figure 3-2 *can* be used for the M method, but requires additional steps. First for a case involving a single specification limit, n and k are determined from the nomograph as before. Figure 3-3 is then used to convert the value k into a value M.

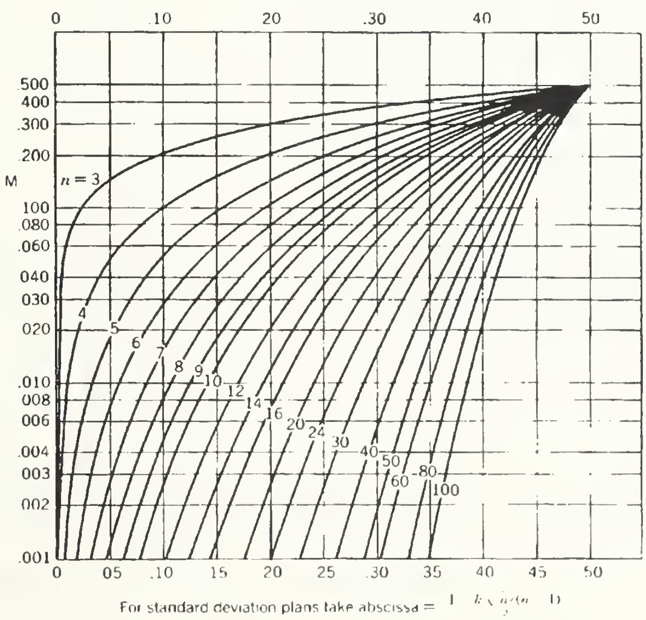

Figure 3-3 Converting k to M [Montgomery, 1991, p 629]

Figure 3-3 is entered from the X axis, but to obtain that value it must first be calculated from Equation 3-2 .

$$X = \frac{1 - \frac{k\sqrt{n}}{(n-1)}}{2}$$

Equation 3-2

where: X = abscissa on Figure 3-3
k = value obtained from Figure 3-2
n = sample size obtained from Figure 3-2

Reading up from the X axis to the intersection with the sample size, M is then read horizontally from the Y axis. As long as the fraction defective is ≤ M, the lot is accepted. But to figure the fraction defective in terms of Z requires yet another step. Here Z, calculated from Equation 3-1 is used to enter Figure 3-4.

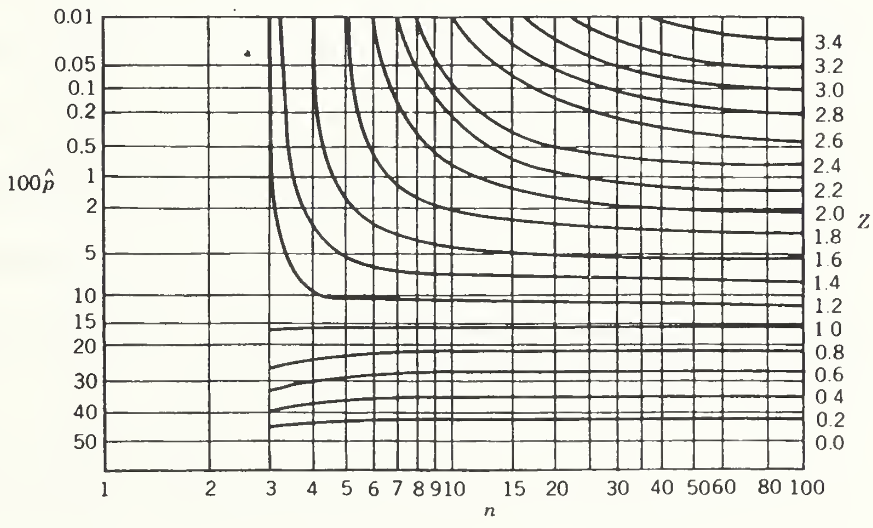

Figure 3-4 Fraction Defective from Z [Montgomery, 1991, p 629]

The fraction defective is then read horizontally on the left vertical axis from the intersection of Z read from the right vertical axis and n up from the horizontal axis.

Again, as long as the fraction defective is ≤ M, the lot is accepted. If Figure 3-2 is to be used for a double specification limit, M method plan, k and n are found as before, then M from Figure 3-3 and Z_{LSL} and Z_{USL} from Equation 3-1. Both Z's are converted to fraction defective from Figure 3-4 and added together. If the two added together are ≤ M, the lot is accepted [Montgomery, 1991, pp 628-630]. Note that it is possible for Z_{LSL} and Z_{USL} to vary, and still result in the same fraction defective.

MIL-STD 414

Following World War II, the Department of Defense began to consolidate the sampling plans that had been developed during the war. MIL-STD-414, acceptance sampling by variables, was introduced in 1957 as an alternative to MIL-STD-105, acceptance sampling by attributes [Duncan, 1986, p 290], [Montgomery, 1991, p 630]. It was originally intended for use in Government procurement, supply and storage, and maintenance inspection operations where a single quality characteristic can be measured. The standard is set up for expressing quality in terms of percent defective, but can be easily modified for just the opposite, which would be percent within limits. The underlying assumption in developing this plan was that the single quality characteristic measured in a random sample is normally distributed. MIL-STD-414 can still be used in nonnormal situations, however the risks involved will be different than those indicated on the operating characteristic curves included in the standard [Duncan, 1986, p 301].

Advantages/Considerations

Compared to attributes sampling plans, like MIL-STD-105, the advantage is that smaller sample sizes can be used for the same level of confidence. Of course the trade off is that for any given sampling plan, it will likely be more expensive to quantitatively measure a single characteristic against a standard, rather than determine a simple pass or fail as in the attributes sampling plan. Therefore early in developing a sampling plan, a quantitative decision must be made to determine which approach is more cost effective; relatively small samples and meticulous measurements, or large samples with simple pass/fail measurements.

In addition, if a standard such as MIL-STD-414 is indiscriminately used, and applied in a situation where the data is not normally distributed, the result will be an inability to accurately predict the risks of accepting a product at varying levels of process quality. In other words, the operating characteristic curves included in the standard begin to lose applicability with increased skewness or kurtosis. Skewness is a measure of how equally distributed data is around the mean, and kurtosis is a measure of peakedness [Blank, 1980, pp 67, 70].

MIL-STD 414 Sections

MIL-STD-414 is divided into four sections, A through D. Section A gives a general description of terms used, a method for classifying defects as "Critical, Major, or Minor", the range of acceptable quality levels used in the standard (0.04-15%), acceptability criterion, and sample selection [MIL-STD-414, pp 1-3]. Section B covers

sampling plans where the variability of the process is unknown, and the standard deviation is used to determine percent defective. This is done through one of two methods, Form 1 or Form 2, which will be described shortly. Section C covers sampling plans where the variability is unknown, but uses a range method in lieu of standard deviation to determine percent defective. The range method is not commonly used today because the results of this process are not as meaningful as when using the standard deviation method. The process was developed because it is mathematically easier to manipulate. With today's calculators and computers there remains little justification for using this method. Section D is used when the process variability is known. The advantage here is that if the process with its inherent variability is known well enough, smaller sample sizes can be used to determine lot quality with the same confidence of the plans in sections B and C. This results in even cheaper sampling.

All methods, B through D, provide for sampling plans based on single, and double specification limits. A single specification limit would be a plan with the criterion that the sample would be either ≤, or ≥ a single value such as for checking compaction of asphalt concrete. A double specification limit is used when a range of values is acceptable, but falling below or above that range is not, as in specifying an asphalt content.

In sections B, C, and D, a choice of either Form 1 or Form 2 is available depending on the circumstances. Other statistics references, such as Montgomery and Duncan, which discuss acceptance sampling procedures, describe these approaches as k and M methods, or Procedures 1 and 2. Form 1, Procedure 1, and the k method are the same. This technique works by specifying a minimum distance, k, from the mean of the sample

data to the value at which the plan wishes to reject material (single specification limit) in terms of a number of standard deviations. If that number is greater than k, the lot is accepted. Form 2, Procedure 2, and the M method are also the same. In this technique, instead of specifying a distance from the mean, a maximum area under the normally distributed curve is not to be exceeded. It can be readily seen that the M method can be used for either single specification or double specification limit plans, whereas the k method is suited only for single specification limit plans.

Methods

After determining the acceptable quality level (AQL) for the characteristic to be measured, and how large the lot will be, Tables A-1 and A-2 in MIL-STD-414 can then be used to determine a specific AQL, and sample size code letter. With these two pieces of information, the tables in Sections B, C, and D can be accessed for sample size and acceptance numbers. The acceptance criterion will either be a distance (k) from the mean, or an area expressed as a percentage (M) as described above. Unless circumstances dictate otherwise, normal inspection is always used first. The plan allows for normal, tightened, and reduced inspection. It should be noted that if MIL-STD-414 is to be used at all, it should be used as closely as possible to the way it was intended. The reason is that confidence in the tables and operating characteristic curves is diminished, or eliminated when the plan is not used as intended. In other words, if reduced sampling is indicated, there should be reduced sampling. The opposite holds true also. If there is more than one quality characteristic, there should be a comparable number of inspection plans.

As mentioned earlier, the primary advantage of a variables sampling plan is that it requires a smaller sample than an attributes plan for the same operating characteristic curve. One of the major disadvantages is that it is necessary to have a separate plan for each quality characteristic inspected. For example, if an item were inspected for three quality characteristics, it would require three separate variables inspection plans.

Impact of Very Low AQL's

Another primary disadvantage of variables sampling plans is that if the process is nonnormal, and the sample size is very small, the probability computations could be seriously affected. All variables sampling plans use the mean and standard deviation to estimate the fraction nonconforming. It is readily seen by studying Figure 2-6 and Figure 2-7, that if the AQL is very small, it would be relatively far out into the tail(s) of the distribution. If the distribution is nonnormal, i.e. peaked or skewed, the effects would be more noticeable on the tails. An example from Duncan may illustrate this more clearly:

> ...if the mean of a normal process or lot lies three standard deviations below a single upper specification limit, it will have no more than 0.00135 nonconforming. On the other hand, if in a nonnormal process or lot with considerable skewness and/or kurtosis say with γ_1=1.00 and γ_2=1.5, the mean lies three standard deviations below the specification limit, possibly 0.01000 of the items might be nonconforming or seven times that for a normal distribution [Duncan, 1986, p 256].

Figure 3-5 below shows how nonnormal distributions affect the tail area as compared to normal.

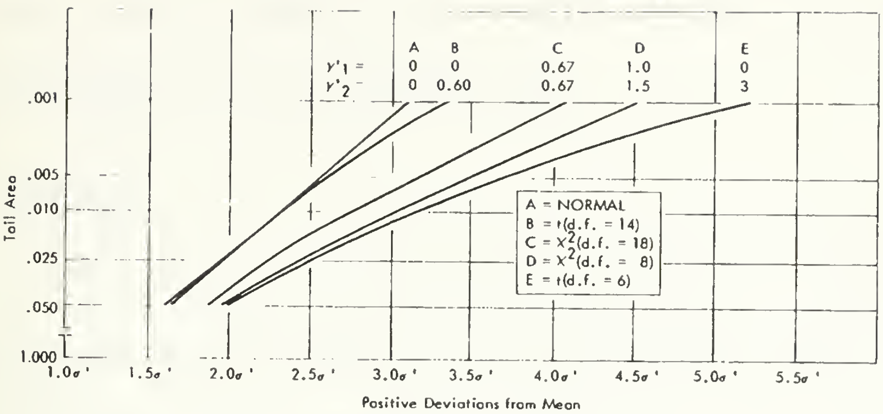

Figure 3-5 Effect of Small AQL on Distribution Tail Area [Duncan, 1986, p 256]

MIL-STD 105

The focus of this report centers on variables sampling, and therefore plans similar to MIL-STD-414. The discussion would not be complete, however, without a brief description of MIL-STD-105, attributes sampling. It too was developed during World War II, and the first version, MIL-STD-105A was issued in 1950 [Montgomery, 1991, p 585]. The latest version is MIL-STD-105E. It is a collection of sampling schemes including single sampling, double sampling, and multiple sampling. For each of these schemes, there are provisions for normal, tightened and reduced inspection. If the plan is a percent defective plan, the AQL's range from 0.10% to 10%. If instead it is a defects per unit plan, there are ten different AQL's up to 1000 defects per 100 units [Montgomery, 1991, p 586] [MIL-STD-105].

As in MIL-STD-414, sample size is determined by lot size and the level of inspection. In addition there are four special inspection levels, S1 through S4 that are

used for very small samples, but only in cases where large risks can be tolerated [Montgomery, 1991, p 603] [MIL-STD-105].

Advantages

Although subject to some controversy, as the sample size increases, the probability of acceptance goes up for AQL work. The effect is that there is less chance of rejecting a large lot, and produces a steeper OC Curve [Montgomery, 1991, p 607].

Disadvantages

The standard emphasizes only the producer's risk end of the OC Curve. The only way to control the discriminatory power of the curve is by choosing sample size, and not all sample sizes are available for use. As the lot size increases, so does the sample size, but at a decreasing rate after n=80 [Montgomery, 1991, p 605].

Generally, larger sample sizes are required for the same level of confidence for attributes sampling plans as compared to variables sampling plans.

Developing Formulas for Pay Curves

Chapter 4, describes the two common methods for employing an adjusted pay schedule for a statistical specification. Obviously a pay formula could be any number or types of equations. Its intent is to provide a smooth transition from bonus pay at superior quality, down to substantially reduced pay at the rejectable quality level. If sufficient study has been made in preparing a statistical specification, the designer will have a good feel for the needed pay reductions at lower quality levels to cover the costs of earlier than programmed repairs. Presuming those costs have been quantified to reflect a withheld

amount for a specific level of quality, it would be straightforward to plot the data and graphically examine the pay trend from superior to poor quality. It might be expected that the curve drawn through the points on this graph would mirror the OC Curve. After all, the OC Curve represents the discriminating power of the acceptance plan, so the pay curve would reflect varying pay factors over the same range of quality. This, however, is not the case with the pay factor tables found in the FP-85 "Standard Specifications for Construction of Roads and Bridges on Federal Highway Projects", and WSDOT's 1994 Standard Specification. If the "steps" are ignored, these pay tables show a general downward curve from bonus to rejection, with Figure 3-6 illustrating this process.

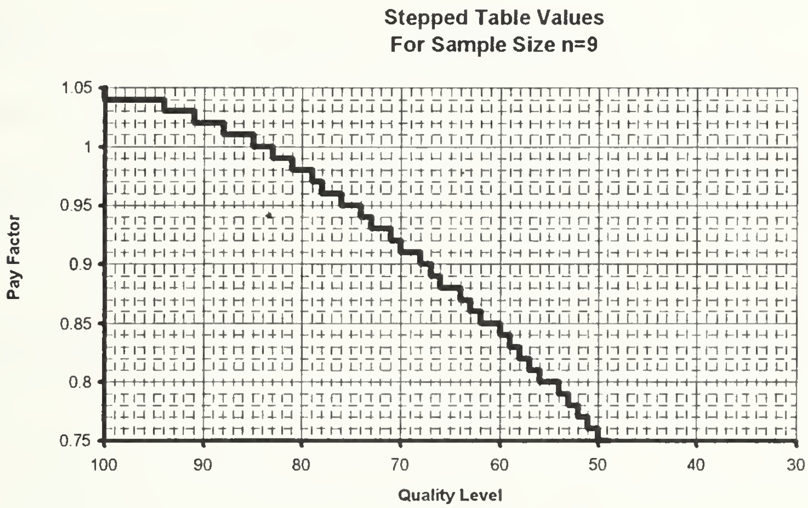

Figure 3-6 Stepped Pay Factors

The larger the sample, the "flatter" the curve becomes, eventually coming close to a straight line from bonus to rejection for large sample sizes. But the "steps" cannot be ignored since they demonstrate a critical aspect of how the pay tables are implemented.

Why the Nonlinear Formula was Used

Even if a specification using a pay table similar to the ones in the FP-85, or WSDOT Standard Specification has appeared to perform as it was intended, it may still be desirable to replace the tables with a pay formula. This is mainly because a plan designer, or contract administrator may wish to do away with potential conflicts with the contractors over missing a higher pay increment because of the "steps". It should be noted that the pay steps in WSDOT's specification are relatively small and therefore may not be as likely to create conflict than if the pay steps were large.

Weed, of the New Jersey Department of Transportation has developed a program called OCPLOT which is designed as a tool for plan designers to do a "what if" analysis on their pay formulas to see if it will perform as desired. The limitations are that only two general pay equation formats, called linear and nonlinear in the program, are available for analysis. The linear equation is $PF = A - B(PD)$, where PF is the pay factor, A is the bonus that would be paid when there are zero defects, B is a constant, and PD is the percent defective. The nonlinear equation is $PF = A - B(PD)^C$, where B and C are both constants. It should be noted that OCPLOT also offers to the user the opportunity to use the equivalent percent within limits formulas instead of percent defective. Because OCPLOT was used to evaluate the pay factor tables in both the FP-85, and WSDOT's 1994 Standard Specification, only the nonlinear equation was used to approximate what an alternative pay formula curve would look like compared to the stepped values. Appendix C shows the behavior of the pay tables compared to the curve generated from the nonlinear pay equation used in OCPLOT. The nonlinear formulas were developed by

using 105 for A, since perfect quality, or zero defects, has a pay factor of 1.05. Next the quality level at a pay factor of 1.00 and 0.75, was read from the table and subtracted from 100 for percent defective. This resulted in two equations and two unknowns. From here, algebra was used to determine B and C for each sample size. As evidenced by Appendix C, the nonlinear formula is a good approximation of the stepped table values. The pay formula derivations are included in Appendix B. OCPLOT will be discussed in more detail later.

Chapter 4

WSDOT SAMPLING PLAN

WSDOT's asphalt concrete QA specification is similar to MIL-STD-414 in that it uses the variability unknown, double specification limit, standard deviation method, for estimating lot quality. Randomly selected sample data is used to compute the mean and standard deviation, then Quality Indexes are computed for entering Table 1 on page 1-34 in the 1994 Standard Specification, to determine percents within upper and lower specification limits. For a double specification limit plan, the percents within limits are added together, and then 100 is subtracted. The resulting quality level is then used to determine a final pay factor which is subsequently used in formulas for the job mix compliance incentive factor, and compaction incentive price adjustment factor. These factors are ultimately used to calculate the final adjustment to the contractor's bid price per ton of asphalt concrete. Refer to WA-RD 326.1, pages 8-15 for an example [Markey, et. al., 1994, pp 8-15].

Departure from MIL-STD 414

Beyond calculating mean, standard deviation, and quality indexes, the WSDOT Standard Specification departs from MIL-STD-414. Table 1 of the 1994 Standard Specification is the WSDOT equivalent of MIL-STD-414's Table B-5. The fact that WSDOT uses percent within limits instead of MIL-STD-414's percent defective is not significant. Of limited significance is that WSDOT Table 1 and MIL-STD-414 Table B-5 do not use the same sample size categories, but as was mentioned in the introduction, the

specification designers' intent was not to exactly mirror MIL-STD-414. The resulting estimates of percents defective for some sample sizes will then invariably be different than if MIL-STD-414 were used over WSDOT's Standard Specification.

MIL-STD-414 uses a two step process to determine the appropriate sample size for a lot. First Table A-2 is entered using the lot size. Assuming "normal" inspection, which is inspection level IV, the table gives a sample size code letter. Then for double specification limit, normal or tightened inspection, Table B-3 is entered for the appropriate sample size and maximum percent defective, M, for the chosen AQL. If sampling other than "normal" is needed, Table A-2 gives different sample size code letters, and Table B-3 can be read from the bottom for tightened inspection. Here it is important to note that MIL-STD-414 is very specific about the sample size needed for a given lot.

By contrast, WSDOT's Standard Specification typically results in at least 5 sublots of about 500 tons each for a minimum of 5 samples. Specifically it states:

> ...For the purpose of acceptance sampling and testing, a lot is defined as the total quantity of material or work produced for each job mix formula (JMF), placed and represented by randomly selected samples tested for acceptance. All of the test results obtained from the acceptance samples shall be evaluated collectively and shall constitute a lot. Only one lot per JMF will be expected to occur...
>
> ...The quantity represented by each sample will constitute a sublot. Sampling and testing for statistical acceptance shall be performed on a random basis at the frequency of one sample per sublot, with a minimum of five sublots per class of mix. Sublot size shall be determined to the nearest 100 tons to provide not less than five uniform sized sublots, based on proposal quantities, with a maximum sublot of 800 tons.
>
> Sampling and testing for nonstatistical acceptance shall be performed on a random basis at a minimum frequency of one sample for each sublot of 400 tons or each day's production, whichever is least [Standard, 1994, 5-04.3(8)A, p 5-22]

This demonstrates the difference between MIL-STD-414's scheme for determining sample size and WSDOT's Standard Specification. There is no clear link between the two standards for determining the number of samples needed per lot because WSDOT's sampling methodology is based on the non-central t-distribution which takes into account the effects of small sample sizes.

Pay Factor Tables/Curves

Earlier the concept of accepting material or work that was below the acceptable quality level, but above the rejectable quality level, was introduced. The idea is that since the material is not up to the minimum level of quality, but better than rejectable, it should receive less pay. The amount of that reduction is based on the amortized value of work which will be needed to repair or replace the defective material at some point earlier than if it had been of better quality. Or in other words, the necessity for repair or replacement has a real cost associated with an earlier than programmed maintenance schedule. The amount withheld from the contractor is theoretically set aside to cover the costs of the expected premature replacements or repairs. This assumes that the costs associated with poorer quality material or work has been quantified sufficiently to make equitable and realistic adjustments to a contractor's pay.

Description

There are two general approaches to implementing a pay scheme that will pay a bonus for superior quality work, 100% at the acceptable quality level, and reduced pay down to the rejectable quality level. They are by using either a pay factor table, or a pay

formula. Table 4-1 is a sample taken from the FP-85, that is duplicated in WSDOT's 1994 Standard Specification.

Potential Problems

The pay factor tables in the FP-85 and WSDOT Standard Specification have a footnote indicating that if the computed quality level does not exactly match the value in the table for a given sample size, then the next lower pay factor should be used. Figure 3-6 demonstrates graphically how these "stepped" pay functions work. An alternative to such tables is presented below.

Alternatives

The alternative, using a pay formula instead of a table such as Table 4-1, is a formula which makes a smooth progression from bonus pay for superior work, to substantially reduced pay at the RQL. Presumably for the pay curve to operate properly, it must pass through, or very close to 1.00 at the AQL, and the lowest pay factor allowable under contract at the RQL. As will be demonstrated later, the pay scheme must allow for a bonus as well as reduced pay for it to operate properly. The slope of the line should match as closely as possible the reductions in pay that are needed at lower quality levels to sufficiently cover the costs of future repair or replacement as mentioned above. The added advantage of a pay formula over pay tables, is that there can be no dispute over a higher pay factor that might have been missed by only a few hundredths of a point. And there are no steps to dispute because the pay factor equation simply indicates a point on the pay curve somewhere between perfect and rejectable quality.

Table 0-1 Pay Factor Table [Standard, 1994, 1-06.2(2), pp 1-36, 37]

Pay Factor	n-3	n-4	n-5	n-6	n-7	n-8	n-9	n-10 to n-11	n-12 to n-14	n-15 to n-18	n-19 to n-25	n-26 to n-37	n-38 to n-69	n-70 to n-200	n-201 to n-∞
1.05	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1.04	90	91	92	93	93	93	94	94	95	95	96	96	97	97	99
1.03	80	85	87	88	89	90	91	91	92	93	93	94	95	96	97
1.02	75	80	83	85	86	87	88	88	89	90	91	92	93	94	95
1.01	71	77	80	82	84	85	85	86	87	88	89	90	91	93	94
1.00	68	74	78	80	81	82	83	84	85	86	87	89	90	91	93
0.99	66	72	75	77	79	80	81	82	83	85	86	87	88	90	92
0.98	64	70	73	75	77	78	79	80	81	83	84	85	87	88	90
0.97	62	68	71	74	75	77	78	78	80	81	83	84	85	87	89
0.96	60	66	69	72	73	75	76	77	78	80	81	83	84	86	88
0.95	59	64	68	70	72	73	74	75	77	78	80	81	83	85	87
0.94	57	63	66	68	70	72	73	74	75	77	78	80	81	83	86
0.93	56	61	65	67	69	70	71	72	74	75	77	78	80	82	84
0.92	55	60	63	65	67	69	70	71	72	74	75	77	79	81	83
0.91	53	58	62	64	66	67	68	69	71	73	74	76	78	80	82
0.90	52	57	60	63	64	66	67	68	70	71	73	75	76	79	81
0.89	51	55	59	61	63	64	66	67	68	70	72	73	75	77	80
0.88	50	54	57	60	62	63	64	65	67	69	70	72	74	76	79
0.87	48	53	56	58	60	62	63	64	66	67	69	71	73	75	78
0.86	47	51	55	57	59	60	62	63	64	66	68	70	72	74	77
0.85	46	50	53	56	58	59	60	61	63	65	67	69	71	73	76
0.84	45	49	52	55	56	58	59	60	62	64	65	67	69	72	75
0.83	44	48	51	53	55	57	58	59	61	63	64	66	68	71	74
0.82	42	46	50	52	54	55	57	58	60	61	63	65	67	70	72
0.81	41	45	48	51	53	54	56	57	58	60	62	64	66	69	71
0.80	40	44	47	50	52	53	54	55	57	59	61	63	65	67	70
0.79	38	43	46	48	50	52	53	54	56	58	60	62	64	66	69
0.78	37	41	45	47	49	51	52	53	55	57	59	61	63	65	68
0.77	36	40	43	46	48	50	51	52	54	56	57	60	62	64	67
0.76	34	39	42	45	47	48	50	51	53	55	56	58	61	63	66
0.75	33	38	41	44	46	47	49	50	51	53	55	57	59	62	65

WSDOT 1994 Standard Specification Issues

Although WSDOT's Standard Specification text does not match exactly the FP-85, the acceptance procedures are essentially the same since calculating mean, standard deviation, and quality indexes is the same, and quality level and pay factor tables are identical. MIL-STD-414 uses mean, standard deviation, and quality indexes to estimate lot quality, which is an accepted standard. The potential problems arise from the FP-85, which WSDOT uses as a source, where it uses a misleading statement relating quality level, pay factors, and their relationship to risk. It is incorrect to assume that acceptance plan OC Curves, and pay curves are the same thing unless specifically linked as in OCPLOT's computer simulation, or NONCENTT [Barros, 1982]. Pay curves and OC Curves represent two very distinct, and different aspects of QA methodology. This will be explained in further detail below.

Excerpt from FP-85

The FP-85 makes the following statement in describing acceptance plan behavior, risk, and pay factors:

> Quality Level Analysis is a statistical procedure for estimating the percent compliance to a specification and is affected by shifts in the arithmetic mean ($\overline{X}$) and by the sample standard deviation (s). Analysis of each test parameter will be based on an Acceptable Quality Level (AQL) of 95.0 and a producer's risk of 0.05. AQL may be viewed as the lowest percent of specification material that is acceptable as a process average. The producer's risk is the probability that when the Contractor is producing material exactly at the AQL, the materials will receive less than a 1.00 pay factor [FP-85, 1985, p 46].

The fact that the AQL is 95% and producer's risk, α, is 5% may be misleading without additional explanation. That the two in this case happen to add up to 100% is coincidental, and it should not be assumed that this is a normal aspect to acceptance sampling. As an example, it would be just as correct to say the AQL is 5% defective, and that the producers risk is 5%, which do not add to 100%.

The FP-85 is also making a sweeping assignment of α risk for all quality characteristics the standard may be used to examine but no specific statement is made concerning the consumer, or β, risk. In other words, according to the statement above, it is assigning a single level of risk to all characteristics being measured regardless of criticality. As was mentioned earlier, at least two points on the OC Curve are needed to design the plan. If only the α risk is specified, there is no way to "nail down" the other end of the OC Curve at the RQL for a given sample size given the methods and explanations of statistical sampling thus far. NONCENTT will allow "nailing down" the RQL end of the curve provided sample size is allowed to adjust, or "float" accordingly. The FP-85 also does not mention when it would be appropriate to vary the α and β risks depending on the criticality of the characteristic. For example, is it more important to control fines passing the Number 200 sieve more closely than those passing the Number 8 sieve?

The excerpt above also implies that the contractor is to use the 95% AQL as a specification standard; however, the contractor should apply whatever quality control is necessary and economical to his process to maximize pay. This should assure that the

quality will be equal or better than the AQL. The AQL is simply a tool for the owner, not the contractor, to make an informed decision concerning lot acceptance.

The last sentence in the excerpt implies a situation the plan designers most certainly do not want. That the contractor suffers a 5% risk of receiving a pay factor of less than 1.00 if he is producing exactly at the AQL is also saying that 95% of all pay factors assigned to the contractor have to be greater than 1.00. This is illustrated in Figure 4-1 below.

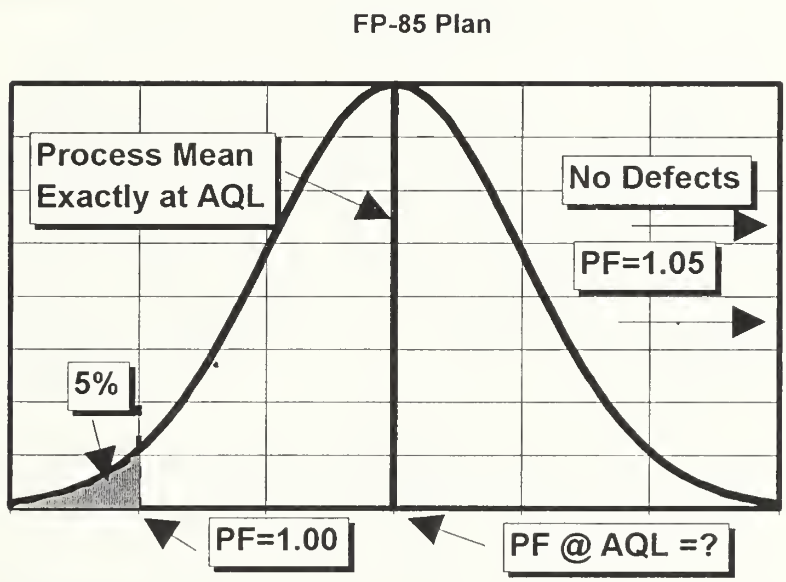

Figure 4-1 Graphical Representation of 5% Risk of Pay Factor < 1.00

Despite the unwanted situation where 95% of the pay factors will be greater than 1.00 for work at the AQL, is that it is very difficult to determine what the average pay factor ends up actually being. Weed has not only noted this problem, but has developed a program, (OCPLOT-described earlier), that will quantify it. After inputting certain plan parameters, including the pay formula, the program enables the plan designer to determine what the

average pay factor will be over a range of quality including the plan's AQL. After manipulating the information from the FP-85 and WSDOT's pay factor tables, OCPLOT determined that the pay tables pay a bonus at all sample sizes, up to and over 104%, for material at the AQL. WSDOT believes that because of this, the contractors factor the bonus into their bids, thereby holding contract bid prices either relatively flat or slightly lower over time. Presumably what the plan designers had really intended is illustrated in Figure 4-2, that if the process is exactly at the AQL, the pay factor should instead be 1.00, and that the contractor is at 5% risk that the AQL work might be rejected.

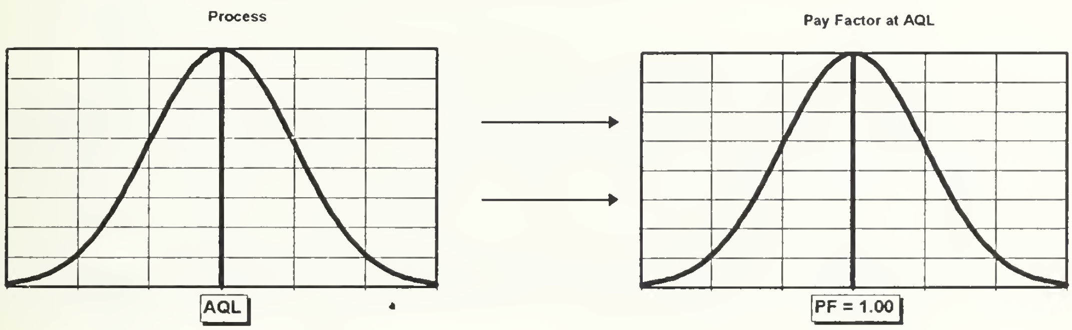

Figure 4-2 Process Exactly at AQL, therefore Pay Factor = 1.00

The plan designer must recognize that for a plan to operate properly, it will be paying a bonus 50% of the time, and a penalty 50% of the time when the process is operating exactly at the AQL. It is not possible to pay a pay factor of 1.00 on average, at the AQL under any other circumstances. This is illustrated in Figure 4-3.

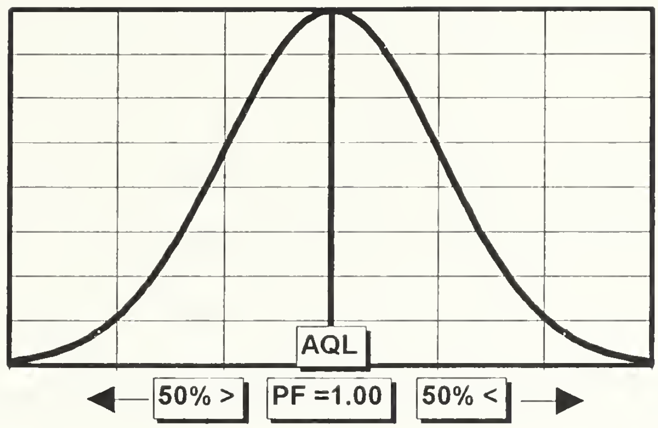

Figure 4-3 Reality of Pay Factors at AQL

The acceptance plan must pay a bonus for superior quality work for it to operate properly. It is not reasonable to expect a contractor to be able to produce completely defect free material and work, nor is it possible for a sampling plan to never make errors in determining lot quality. Recognizing now how a contractor's process can be visualized, Figure 4-4 demonstrates what happens when there is no bonus provision.

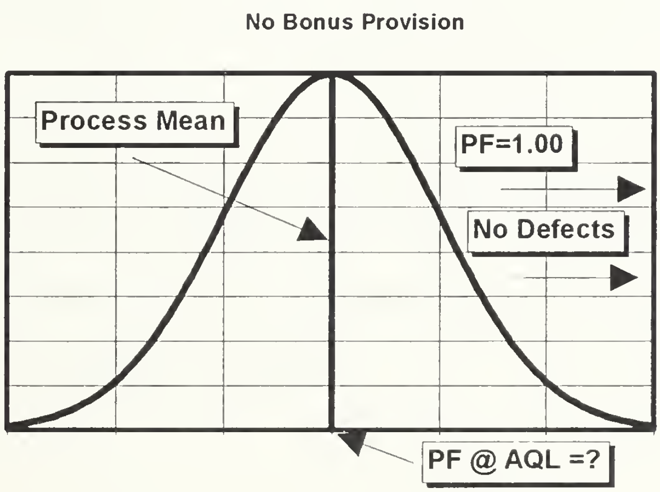

Figure 4-4 Results of No Bonus Provision

This plan would result in the contractor being unduly penalized since his process would on average pay less than 1.00 for work at the AQL. Again, OCPLOT is a good tool to determine what the average pay factor would be at the AQL.

Note that Figure 4-2 represents two related, but very different aspects of acceptance sampling: The contractor's process, and the pay as a result of the sample quality. Also it should be noted that the OC Curve and the pay curve are two related but very different aspects of the acceptance plan. The OC Curve is a graphical representation of the discriminating power of the sampling plan. The pay curve represents the progression of pay factors from a bonus at superior quality, to 1.00 at the AQL, to reduced pay down to the RQL. The pay curve cannot be used to determine the discriminating power of the acceptance plan, nor can it be used to determine risk.

Flatter (Less Discriminating) OC Curves

It should be noted again that the discriminating power of the acceptance plan is markedly affected by the sample size. As the sample size increases, the discriminating power goes up. In other words, the larger the sample, the greater the confidence in its results, which means the probability of accepting substandard material should decrease. For relatively smaller sample sizes, the OC Curve will be flatter over the quality spectrum which means that the chance that good material might be rejected is held to a minimum. This is illustrated in Figure 4-5 below.

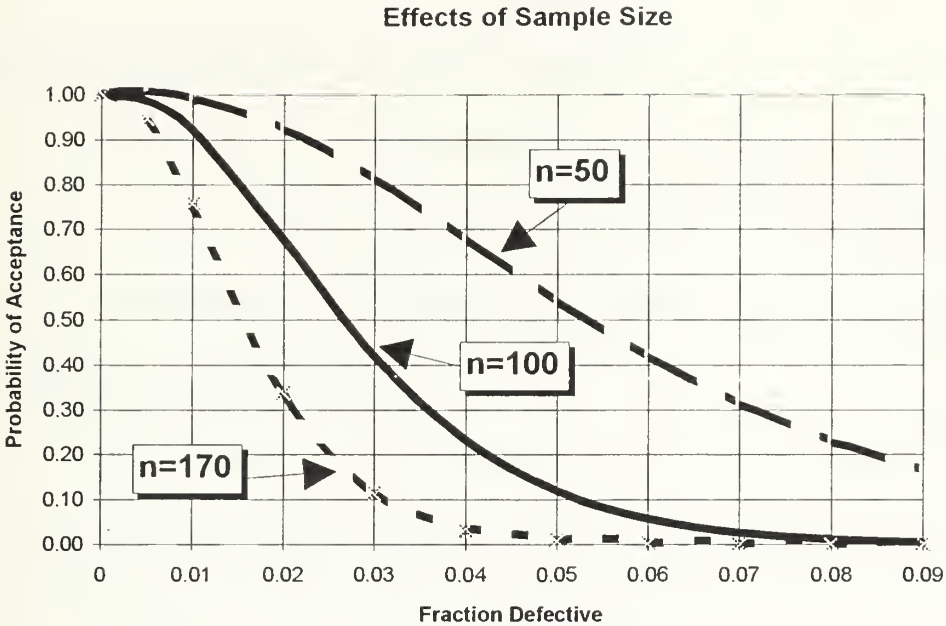

Figure 4-5 Effect of Sample Size on the Discriminating Power of the Plan

Although the curves in Figure 4-5 were created using formulas for an attributes plan, the concept is the same for variables. One of the biggest advantages of a variables plan is that smaller sample sizes can be used for the same level of confidence. It should be noted however, that WSDOT's Standard Specification allows for sample sizes as small as "5 sublots per class of mix."

Chapter 5

New Jersey DOT QA Research and OCPLOT

The New Jersey Department of Transportation began implementing statistical quality assurance specifications in the late 1960's. A brief overview of how their program evolved provides insight into developing a QA specification, and why OCPLOT was developed as a tool to that end.

A better method for describing quality was desired, and with most new procedures has been an evolving process ever since. The concepts of acceptable and unacceptable work were expressed in terms of the average value in relation to the specification limits. However, this method did not take variability into account and it was soon discovered that on average the material could be judged acceptable even though a substantial amount was out of specification. Here the NJDOT realized the importance of controlling variability, thus implementing specifications based on the variables sampling procedures described in MIL-STD-414. The added bonus was that this was a more efficient procedure requiring fewer samples. It was also found that pay equations had benefits over pay tables in that disputes were avoided by eliminating pay "steps". More recently it was discovered that for an acceptance procedure to operate fairly, it would have to provide a bonus provision for reasons cited earlier. In many cases a linear pay equation was sufficient, but to provide adequately low pay at the AQL sometimes also required the bonus to be unusually high. OCPLOT proved to be a valuable tool in developing the DOT's new specifications. To allow the contracting community to become familiar with the new specifications, New Jersey has implemented a policy whereby pay factor deductions are reduced by one half as

the specification is phased in [Weed, 1994]. Weed, 1994, "Development of Air Voids Specifications for Bituminous Concrete" is an excellent example of QA specifications evolution, and methods for implementation.

OCPLOT's approach is to use computer simulation to estimate lot quality. It is important to note here that this is the only approach besides NONCENTT which makes it possible to directly relate the acceptance plan performance and the resulting pay factors generated for varying levels of lot quality.

The first menu following the introductory screens allows the user to input the various features and parameters of the acceptance plan. This includes, but is not limited to whether it is pass/fail or pay adjustment, single or double specification limit, what is the desired pay equation, AQL, RQL, and sample size. The menu items appear in a logical sequence and build upon one another depending on the plan. In other words, if an attributes plan or linear pay equation was desired, a different set of questions would have followed. After the plan parameters have been typed in, the user must select a level of precision desired for the simulation. The simulation process is very computationally intensive, so depending on what type of computer is used makes a difference for which level is selected. A 386 SX-20 laptop computer was used at all three levels. The high precision level took an unacceptably long time to generate all the data sets. A desktop 90 MHz Pentium was also tried, and found that it was fast enough that it did not matter which level was selected. The way the simulation works is that a series of samples of the size designated by the user in the preliminary screen are taken from a randomly generated universe of normally distributed data at each level of quality from the AQL down to the

RQL in increments of approximately 5%. The low precision method generates 200 sample sets, the intermediate level generates 1000 sets, and the high precision level generates 5000 sample sets. For example, if a user had specified a sample size of 5, and had chosen the low level of precision for faster execution, and had specified an AQL of 5% defective and RQL of 25% defective, the simulation would produce 200 randomly selected sample sets of 5 each from a population that was 5% defective, then 10% defective, and so on down to 25% defective. This is illustrated in Figure 5-1 below.

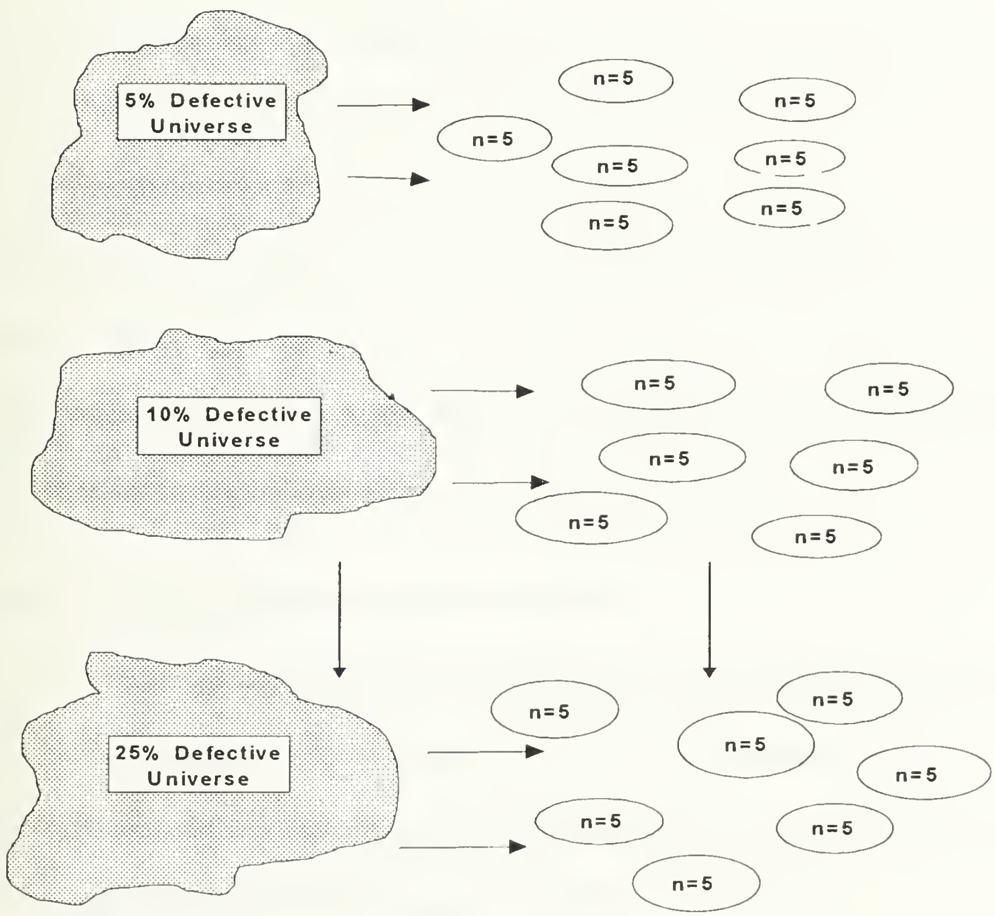

Figure 5-1 OCPLOT Simulation

Each sample of 5 would then be analyzed and the results compared to the boundaries specified in the preliminary screen. This way a "tally" can be made of the number of "items" that fall within each quality level increment between perfect, down to and beyond the RQL. The results are averaged and reveal the expected performance at the AQL and RQL. The corresponding pay factors are matched to these levels of expected performance. This way an average pay factor can be determined for each level of population quality. Since the plan designer is generally interested in what happens at the AQL and RQL, separate screens, among other things, give detailed information showing performance, and pay factor histograms, and an operating characteristic curve showing the relationship between quality and expected pay factor. It is important to note that this operating characteristic curve cannot be equated with those discussed in the rest of this paper. The reason this curve can be called an OC Curve even though it shows expected pay instead of probability of acceptance, is because the simulation directly links plan performance and corresponding pay at that performance level.

Uses for Developing Acceptance Plans

OCPLOT is a powerful tool available for both plan designers and administrators. It allows not only the opportunity to predict how well a plan will perform with respect to pay and quality level, but it also allows a straightforward analysis of existing plans. This is how the FP-85 and WSDOT's pay factor tables were analyzed.

The program does not allow the user to directly assign a desired α and β risk. The only way the OC Curve can be manipulated is through the pay equation, sample size, and

AQL/RQL. In other words, the program will allow the user to modify the pay equation and other parameters so that the plan may pay a bonus for superior quality, exactly 1.00 at the AQL and reduced pay from there, but it does not report what the risk is to the contractor or agency at the AQL or RQL.

OCPLOT can provide an agency the opportunity to avoid unwittingly permitting situations to develop where an acceptance plan pays too much on average at the AQL, or just as importantly, where it is unduly harsh on contractors. The power of OCPLOT lies in its ease of use, its analytical power, and its ability to bring substance to a more esoteric part of applied statistics.

Regression a Potential Tool for Developing Pay Curves

If an agency has determined that a pay scheme using tables instead of a pay formula has worked well in the past, it may be desirable not to change to using a formula despite its inherent advantages. An alternative would be to use regression to curve fit the pay data for each sample size. This way the mechanics of the pay table would be preserved, and the advantages of using a pay formula would be added. Spreadsheet programs such as Excel have built-in data analysis tools that make this an easy task and can be made to display the formula and R^2 value. The only disadvantage to this technique is that it is likely regression will provide a formula that would not allow analysis using OCPLOT. If it is desired to use OCPLOT for analysis, the plan designer has no choice but to use OCPLOT's linear or nonlinear formats. The process is simply to use the percent defective, or percent within limits at the AQL and RQL with their corresponding

pay factors, and solve for the variables using two equations and two unknowns. This technique is demonstrated in Appendix B.

Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

The original goal of this report was to determine risks assigned to the contractors and WSDOT in their asphalt QA specifications, and examine the pay factors used by WSDOT in its sampling plan. By doing so the study provided insight into statistically based sampling plans including pitfalls that may be encountered..

Determination of Risk for WSDOT's 1994 Specification

After determining the statistical model which best fits WSDOT's specifications, and comparing the differences in sampling to MIL-STD-414, it was concluded that it is not possible to accurately determine WSDOT's risk using MIL-STD-414's OC Curves and associated data distributions. By the specification, the contractor's risk was presumably fixed at 5% for all quality characteristics. MIL-STD-414 includes a battery of OC Curves which can be used to determine exactly what the risks would be for different sample sizes and levels of inspection, but WSDOT's specification departs enough from this standard that the OC Curves no longer apply. As was found late in the study, this was by design. The primary reasons for the differences are because WSDOT uses different sample size categories than MIL-STD-414 for determining quality level (which in reality is probably a minor difference), and especially because the sample size is determined differently than MIL-STD-414. The importance of sample size is emphasized in Figure 4-5. Although WSDOT's Standard Specification does not mirror the language of the FP-85, the fact that it uses the same quality and pay tables indicates the plan design is the same.

That FP-85 only indicates a seller's risk, also contributes to the uncertainty of what the buyer's or agency's risk would be for varying sample sizes. As was discussed earlier, to properly design an OC Curve, two points, usually the AQL and RQL, are needed.

At first it appears the risks could be determined directly from the hypergeometric formula, but this is not the case. Although it is true that the way a lot is defined by WSDOT accurately fits the hypergeometric model, the data collected is continuous, not discrete. That effectively eliminates the opportunity to use the hypergeometric formula to "solve for risk". Again, it is worth mentioning that the material used for testing one sample, in proportion to the lot size, is in all practicality like sampling from an infinite universe, with the results closely approximating a normal distribution.

For a single specification limit plan, it might appear to be possible to "work backwards" using the pay tables, and the nomograph. But this is not possible because the pay table does not indicate at a pay factor of 1.00 what the true acceptable quality level is. This is because the pay tables are based on the non-central t-distribution which compensates for small sample sizes in determining lot quality. If it was assumed that the AQL could be read directly from the pay tables, there would be a different AQL for each sample size which is in direct conflict with the statement made in the FP-85. Virtually all the quality characteristics examined in WSDOT's specification are double specification limit items which would require the use of the M method. The nomograph uses fraction defective, and probability of acceptance at the AQL and RQL to determine n and k. The k method essentially only works for single specification limit plans. As pointed out earlier, the M method can be used for either single specification or double specification limit plans

since it uses area under the curve not to exceed, rather than minimum distance between sample mean and specification limit. Figure 2-7 in Chapter 2 indicates that the area under the curve beyond the upper and lower specification limits can shift but still sum to the same quantity. This means that there is an infinite number of combinations for material that may be out of specification outside the upper and lower limits which results in a band of OC Curves [Duncan, 1986, pp 282, 283]. Consequently it would be impossible to work backwards from the information contained in WA-RD 326.1 to determine risk. There is another formula for probability of acceptance for a variables sampling plan [Duncan, Equations 12.1 and 12.2, pp 276-279] but again this is for the k method only and requires that n and k be known. n can be read directly from the pay factor table, but finding k is still not possible.

The NONCENTT computer program remains the best alternative for determining the risks in WSDOT's specification. It is based on the non-central t distribution which compensates for errors in determining lot quality based on small sample sizes. It provides a quantitative means for determining how much more out of specification material can be allowed relative to the sample size, while still ensuring that the lot is of some minimum quality. This is clearly seen in WSDOT's pay factor table where the allowable percent defective is larger for small samples, and decreases as the sample size gets larger for each pay factor. As the number of samples approaches infinity, the allowable out of specification material will approach the AQL for a pay factor of 1.00. Because NONCENTT was not discovered until late in the research, it was not used.

How Nomographs May be Used

The nomographs in Chapter 3, Figure 3-1 and Figure 3-2, are designed as a short cut for a plan designer. Beginning with the fractions defective desired at the AQL and RQL for predetermined levels of risk, or probabilities of acceptance, the nomographs will yield a sampling plan with a given sample size n and acceptance number c, or sample size n and minimum k respectively. Only if appropriate information is known such as fractions defective, sample size and k, can the nomograph be used in reverse to find risk, and then only as long as the plan adhered to accepted methodology such as in MIL-STD-414. Otherwise it may yield two lines that do not intersect inside the nomograph.

WSDOT Pay Factors

Using OCPLOT to analyze the pay factors found in the WSDOT specification indicated that on average Washington is paying more than it should for AQL work. For sample sizes smaller than 20, the pay factor averaged about 104% for AQL work, and 103% to 101% for samples up to and over 200. This was assuming that AQL work allowed 5% defective for all quality characteristics, and that the shape of the pay curve was dictated by the range of quality levels shown in the pay factor table.

Development Standards

There are many alternatives available for statistical sampling that a plan designer may use. The best approach to understanding statistical sampling is to study it from more than one source such as Duncan, Montgomery, and Blank. It is useful to see more than one author's perspective because certain aspects which are unclear in one reference, may

be explained more easily in another. MIL-STD-414 and MIL-STD-105 provide a variables and attributes sampling approach, respectively. Special circumstances may dictate that using one or both of these standards may be inappropriate or expensive. If the designer follows the guidelines in those textbooks named above, it will still be possible to create a plan with specified OC Curves.

In situations where large sample sizes are prohibitively expensive, NONCENTT is worthy of consideration as a tool for creating a viable sampling plan. WSDOT's pay factor tables are based on the non-central t distribution which was not examined in this study.

Use Pay Formula vs Tables

Pay tables are easier to apply than a pay formula, but inaccuracies can occur. The easiest way to avoid such inaccuracies is to use a pay formula. A step by step worked out example in the specification demonstrating how to use it should avoid any confusion in usage. It should maintain the bonus provision, and be developed using OCPLOT to ensure it pays 1.00 at the AQL and 0.75 at the RQL. It may not be possible to achieve these pay factors exactly, but "close" is adequate as long as the pay curve behaves as the plan designer wishes.

Explanation of Sampling Plans

It is human nature to be hesitant about agreeing to terms that are not clearly understood. Statistical sampling plans can be mysterious without an explanation of key concepts such as those presented in this paper. Performance specifications which are

successful in clearly describing the scope of work and site conditions invariably avoid more claims than one which is less clear or less detailed. It stands to reason then, that a specification which intends to use statistical methods for acceptance and payment would benefit from the contractors having a full and clear understanding of the underlying statistical methods and risks. An alternative would be to make available an explanation of statistical sampling and its related concepts in a separate publication which could be referenced in the standard specification.

Recommendations Specific to WSDOT

Since discovering that the pay tables in WSDOT's specification were developed using NONCENTT, it would be worth studying this program to accurately determine, and more fully describe WSDOT's specification risks. Other future work WSDOT may wish to consider is the point of sampling of the asphalt concrete. Though not examined in this study, obtaining samples from the paver hopper versus the truck bed should be considered.

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APPENDIX A: TERMINOLOGY

Acceptance Number: The maximum number of defective items allowed before a lot is rejected. Generally, this term implies a relationship to attributes sampling plans.

Acceptance Plan: A statistically based sampling plan.

α Risk: Also Alpha Risk, Seller's Risk, or Type I error. This is the chance that an owner will reject material that should be accepted.

AQL: Acceptable Quality Level. This is the minimum level of quality that a plan designer wishes to permit, and still pay the contractor 100% of his bid price. This level of quality is an admission that for non-critical characteristics, it is unreasonable to expect zero defects. It is the level of defects that will not have an appreciable impact on the performance, or life of the material.

β Risk: Also Beta Risk, Buyer's Risk, or Type II error. This is the chance that an owner will accept material that should be rejected.

Double Sampling: A procedure where a second sample may be required before a lot can be sentenced. If a sample percent defective is greater than that which would allow unquestioned acceptance, but lower than that which would require outright rejection, a second sample would be taken.

Fraction Defective: Usually expressed as a percentage, this term is typically used to describe how defective the population is from which the lots and samples are drawn.

MIL-STD: Military Standard

OC Curve: Operating Characteristic Curve. This curve is a graphical representation of the discriminating power of a sampling plan. It shows the probability of acceptance of material exhibiting a spectrum of quality from no defects to very defective from which the lots and samples are drawn.

Percent Defective: A term used to describe the quantity defective in a sample.

QA: Quality Assurance. This term implies performance, or outcome based specifications. Many times QA also implies that statistical methods will be employed in contract enforcement.

<u>RQL</u>: Rejectable Quality Level. This is the threshold level of quality below which material will be rejected or replaced. Between the AQL and RQL, the contractor will still be paid, but at a progressively lower factor down to the RQL.

<u>Sentencing</u>: A judgment based on sample results what the disposition of the lot should be.

<u>Single Sampling</u>: One sample of one to thousands of items. In single sampling, a lot is sentenced based on the results of that one sample.

APPENDIX B: PAY FORMULA DERIVATIONS

Sample Size n=3

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.32

PF=0.75 PD=0.67

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(32)^C$$
$$B(32)^C = 5$$
$$LogB + C \bullet Log32 = 0.699$$

$$75 = 105 - B(67)^C$$
$$B(67)^C = 30$$
$$LogB + C \bullet Log67 = 1.477$$

$$0.699 - C \bullet Log32 = 1.477 - C \bullet Log67$$
$$C \bullet Log67 - C \bullet Log32 = 0.778$$
$$C(Log67 - Log32) = 0.778$$

$$C = 2.4244$$
$$B = 0.0011$$
$$\therefore PF = 1.05 - 0.0011(PD)^{2.4244}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=4</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.26

PF=0.75 PD=0.62

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(26)^C$$
$$B(26)^C = 5$$
$$LogB + C \bullet Log26 = 0.699$$

$$75 = 105 - B(62)^C$$
$$B(62)^C = 30$$
$$LogB + C \bullet Log62 = 1.477$$

$$0.699 - C \bullet Log26 = 1.477 - C \bullet Log62$$
$$C \bullet Log62 - C \bullet Log26 = 0.778$$
$$C(Log62 - Log26) = 0.778$$

$$C = 2.0618$$
$$B = 0.0060$$
$$\therefore PF = 1.05 - 0.0060(PD)^{2.0618}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=5

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.22

PF=0.75 PD=0.59

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(22)^C$$
$$B(22)^C = 5$$
$$LogB + C \bullet Log22 = 0.699$$

$$75 = 105 - B(59)^C$$
$$B(59)^C = 30$$
$$LogB + C \bullet Log59 = 1.477$$

$$0.699 - C \bullet Log22 = 1.477 - C \bullet Log59$$
$$C \bullet Log59 - C \bullet Log22 = 0.778$$
$$C(Log59 - Log22) = 0.778$$

$$C = 1.8163$$
$$B = 0.0182$$
$$\therefore PF = 1.05 - 0.0182(PD)^{1.8163}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=6</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.20

PF=0.75 PD=0.56

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(20)^C \qquad 75 = 105 - B(56)^C$$
$$B(20)^C = 5 \qquad B(56)^C = 30$$
$$LogB + C \bullet Log20 = 0.699 \qquad LogB + C \bullet Log56 = 1.477$$

$$0.699 - C \bullet Log20 = 1.477 - C \bullet Log56$$
$$C \bullet Log56 - C \bullet Log20 = 0.778$$
$$C(Log56 - Log20) = 0.778$$

$$C = 1.7402$$
$$B = 0.0272$$
$$\therefore PF = 1.05 - 0.0272(PD)^{1.7402}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=7</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.19

PF=0.75 PD=0.54

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^{C}$

$$100 = 105 - B(19)^{C} \qquad 75 = 105 - B(54)^{C}$$

$$B(19)^{C} = 5 \qquad B(54)^{C} = 30$$

$$LogB + C \bullet Log19 = 0.699 \qquad LogB + C \bullet Log54 = 1.477$$

$$0.699 - C \bullet Log19 = 1.477 - C \bullet Log54$$

$$C \bullet Log54 - C \bullet Log19 = 0.778$$

$$C(Log54 - Log19) = 0.778$$

$$C = 1.7153$$

$$B = 0.0320$$

$$\therefore PF = 1.05 - 0.0320(PD)^{1.7153}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=8</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.18

PF=0.75 PD=0.53

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(18)^C$$
$$B(18)^C = 5$$
$$LogB + C \bullet Log18 = 0.699$$

$$75 = 105 - B(53)^C$$
$$B(53)^C = 30$$
$$LogB + C \bullet Log53 = 1.477$$

$$0.699 - C \bullet Log18 = 1.477 - C \bullet Log53$$
$$C \bullet Log53 - C \bullet Log18 = 0.778$$
$$C(Log53 - Log18) = 0.778$$

$$C = 1.6592$$
$$B = 0.0413$$
$$\therefore PF = 1.05 - 0.0413(PD)^{1.6592}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=9

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.17

PF=0.75 PD=0.51

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(17)^C \qquad 75 = 105 - B(51)^C$$
$$B(17)^C = 5 \qquad B(51)^C = 30$$
$$LogB + C \bullet Log17 = 0.699 \qquad LogB + C \bullet Log51 = 1.477$$

$$0.699 - C \bullet Log17 = 1.477 - C \bullet Log51$$
$$C \bullet Log51 - C \bullet Log17 = 0.778$$
$$C(Log51 - Log17) = 0.778$$

$$C = 1.6309$$
$$B = 0.0492$$
$$\therefore PF = 1.05 - 0.0492(PD)^{1.6309}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=10 to 11</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.16

PF=0.75 PD=0.50

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(16)^C \qquad 75 = 105 - B(50)^C$$
$$B(16)^C = 5 \qquad B(50)^C = 30$$
$$LogB + C \bullet Log16 = 0.699 \qquad LogB + C \bullet Log50 = 1.477$$

$$0.699 - C \bullet Log16 = 1.477 - C \bullet Log50$$
$$C \bullet Log50 - C \bullet Log16 = 0.778$$
$$C(Log50 - Log16) = 0.778$$

$$C = 1.5725$$
$$B = 0.0639$$
$$\therefore PF = 1.05 - 0.0639(PD)^{1.5725}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=12 to 14

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.15

PF=0.75 PD=0.49

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(15)^C$$
$$B(15)^C = 5$$
$$LogB + C \bullet Log15 = 0.699$$

$$75 = 105 - B(49)^C$$
$$B(49)^C = 30$$
$$LogB + C \bullet Log49 = 1.477$$

$$0.699 - C \bullet Log15 = 1.477 - C \bullet Log49$$
$$C \bullet Log49 - C \bullet Log15 = 0.778$$
$$C(Log49 - Log15) = 0.778$$

$$C = 1.5136$$
$$B = 0.0830$$
$$\therefore PF = 1.05 - 0.0830(PD)^{1.5136}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=15 to 18</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.14

PF=0.75 PD=0.47

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(14)^C$$
$$B(14)^C = 5$$
$$LogB + C \bullet Log14 = 0.699$$

$$75 = 105 - B(47)^C$$
$$B(47)^C = 30$$
$$LogB + C \bullet Log47 = 1.477$$

$$0.699 - C \bullet Log14 = 1.477 - C \bullet Log47$$
$$C \bullet Log47 - C \bullet Log14 = 0.778$$
$$C(Log47 - Log14) = 0.778$$

$$C = 1.4795$$
$$B = 0.1008$$
$$\therefore PF = 1.05 - 0.1008(PD)^{1.4795}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=19 to 25

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.13

PF=0.75 PD=0.45

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(13)^C$$
$$B(13)^C = 5$$
$$LogB + C \bullet Log13 = 0.699$$

$$75 = 105 - B(45)^C$$
$$B(45)^C = 30$$
$$LogB + C \bullet Log45 = 1.477$$

$$0.699 - C \bullet Log13 = 1.477 - C \bullet Log45$$
$$C \bullet Log45 - C \bullet Log13 = 0.778$$
$$C(Log45 - Log13) = 0.778$$

$$C = 1.4430$$
$$B = 0.1235$$
$$\therefore PF = 1.05 - 0.1235(PD)^{1.4430}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=26 to 37

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.11

PF=0.75 PD=0.43

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(11)^C$$
$$B(11)^C = 5$$
$$LogB + C \bullet Log11 = 0.699$$

$$75 = 105 - B(43)^C$$
$$B(43)^C = 30$$
$$LogB + C \bullet Log43 = 1.477$$

$$0.699 - C \bullet Log11 = 1.477 - C \bullet Log43$$
$$C \bullet Log43 - C \bullet Log11 = 0.778$$
$$C(Log43 - Log11) = 0.778$$

$$C = 1.3143$$
$$B = 0.2139$$
$$\therefore PF = 1.05 - 0.2139(PD)^{1.3143}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=38 to 69

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.10

PF=0.75 PD=0.41

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(10)^C$$
$$B(10)^C = 5$$
$$LogB + C \bullet Log10 = 0.699$$

$$75 = 105 - B(41)^C$$
$$B(41)^C = 30$$
$$LogB + C \bullet Log41 = 1.477$$

$$0.699 - C \bullet Log10 = 1.477 - C \bullet Log41$$
$$C \bullet Log41 - C \bullet Log10 = 0.778$$
$$C(Log41 - Log10) = 0.778$$

$$C = 1.2699$$
$$B = 0.2689$$
$$\therefore PF = 1.05 - 0.2689(PD)^{1.2699}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

Sample Size n=70 to 200

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.09

PF=0.75 PD=0.38

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(9)^C$$
$$B(9)^C = 5$$
$$Log B + C \bullet Log 9 = 0.699$$

$$75 = 105 - B(38)^C$$
$$B(38)^C = 30$$
$$Log B + C \bullet Log 38 = 1.477$$

$$0.699 - C \bullet Log 9 = 1.477 - C \bullet Log 38$$
$$C \bullet Log 38 - C \bullet Log 9 = 0.778$$
$$C(Log 38 - Log 9) = 0.778$$

$$C = 1.2440$$
$$B = 0.3250$$
$$\therefore PF = 1.05 - 0.3250(PD)^{1.2440}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

<u>Sample Size n=201 to ∞</u>

Percents Defective (100-Quality Level) from Table 4-1 at a Pay Factor of 1.00 and 0.75

PF=1.00 PD=0.07

PF=0.75 PD=0.35

A=1.05

Then using the nonlinear equation from OCPLOT: $PF = A - B(PD)^C$

$$100 = 105 - B(7)^C$$
$$B(7)^C = 5$$
$$LogB + C \bullet Log7 = 0.699$$

$$75 = 105 - B(35)^C$$
$$B(35)^C = 30$$
$$LogB + C \bullet Log35 = 1.477$$

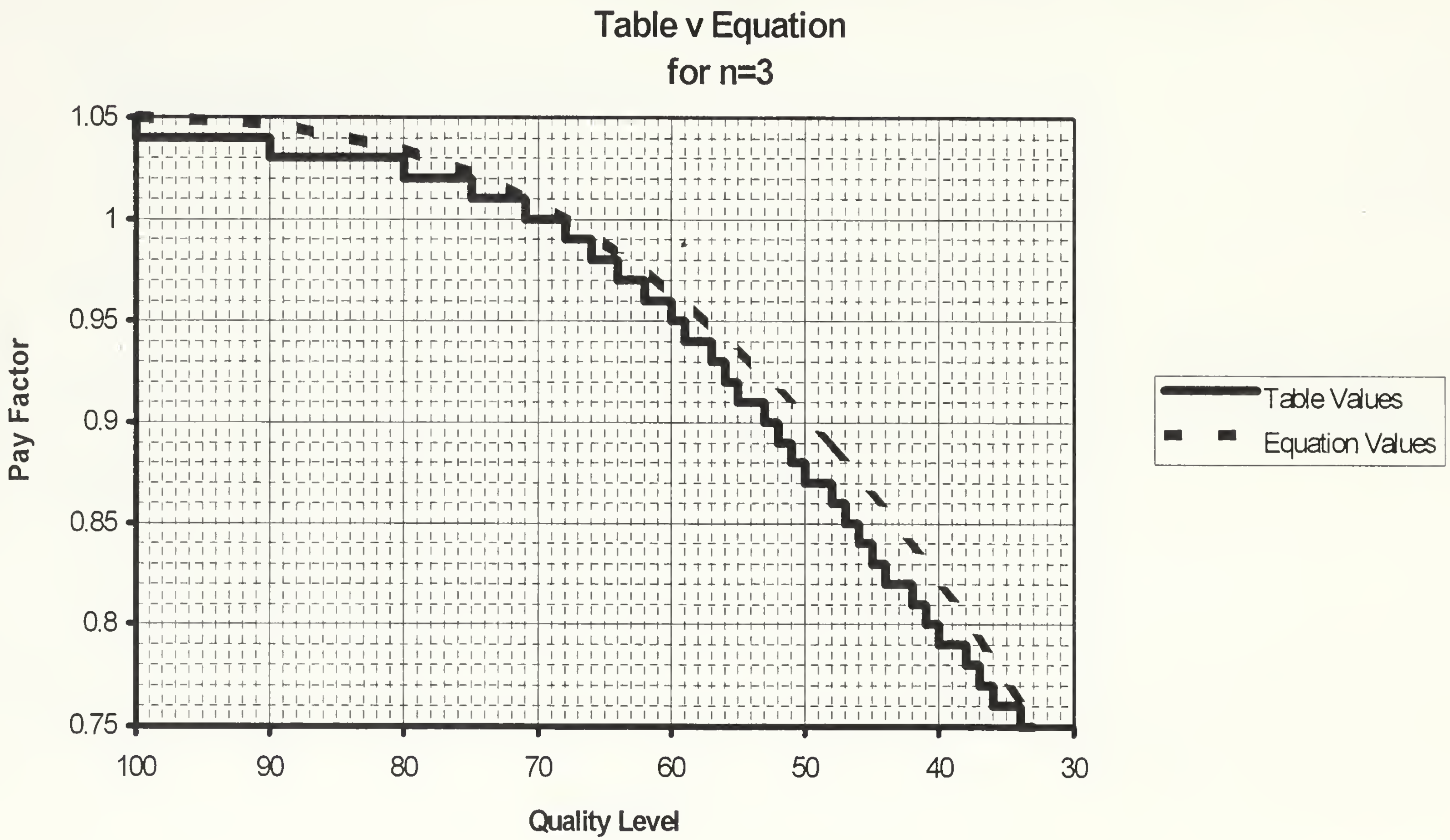
$$0.699 - C \bullet Log7 = 1.477 - C \bullet Log35$$
$$C \bullet Log35 - C \bullet Log7 = 0.778$$
$$C(Log35 - Log7) = 0.778$$

$$C = 1.1133$$
$$B = 0.5730$$
$$\therefore PF = 1.05 - 0.5730(PD)^{1.1133}$$

Note: (1) Decimals are carried through all calculations and are only rounded at the end.

(2) Pay Factors, Percents Defective, and A are multiplied by 100 in the equations.

APPENDIX C: TABLE AND FORMULA PAY CURVES



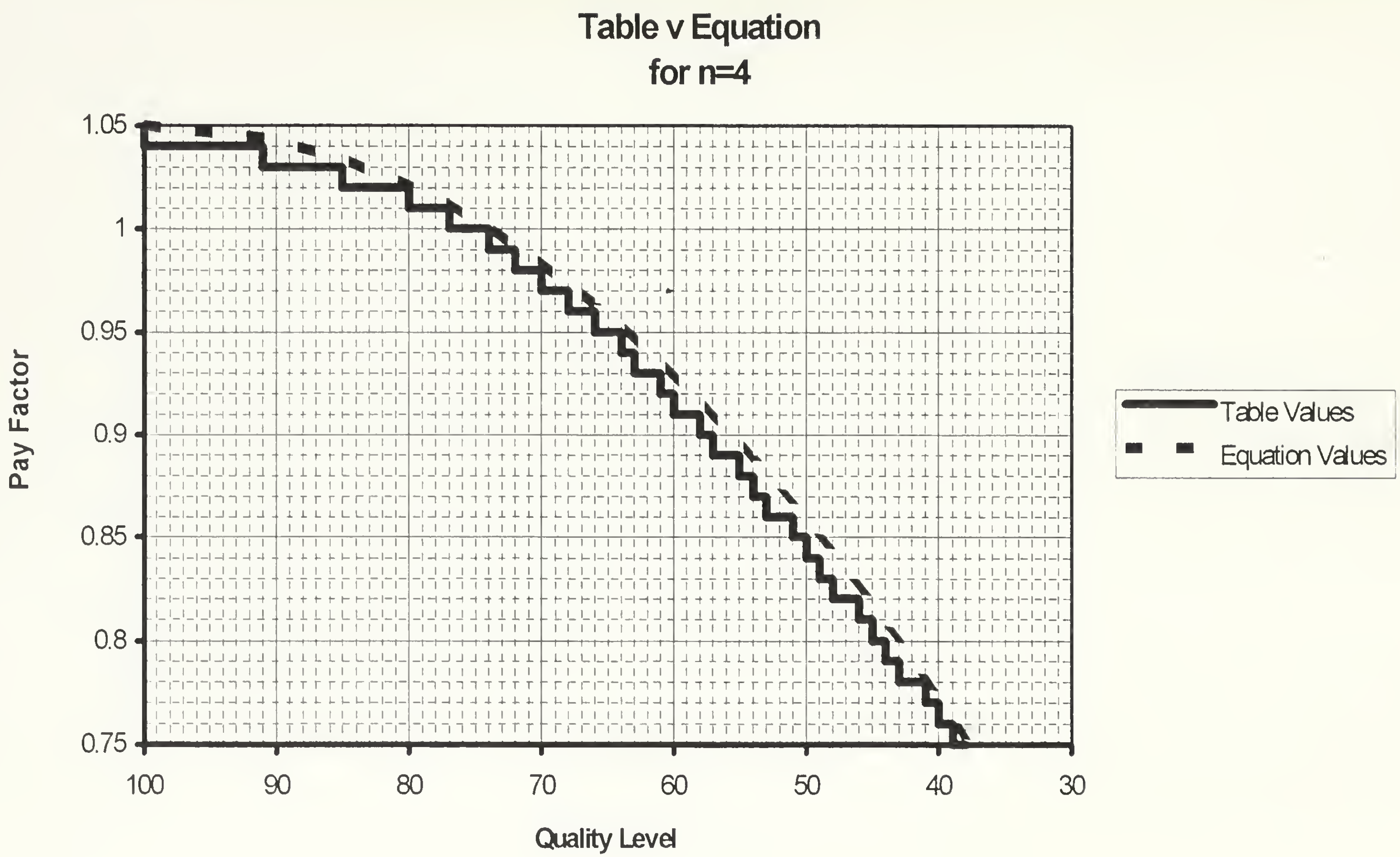


Table v Equation
for n=4
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

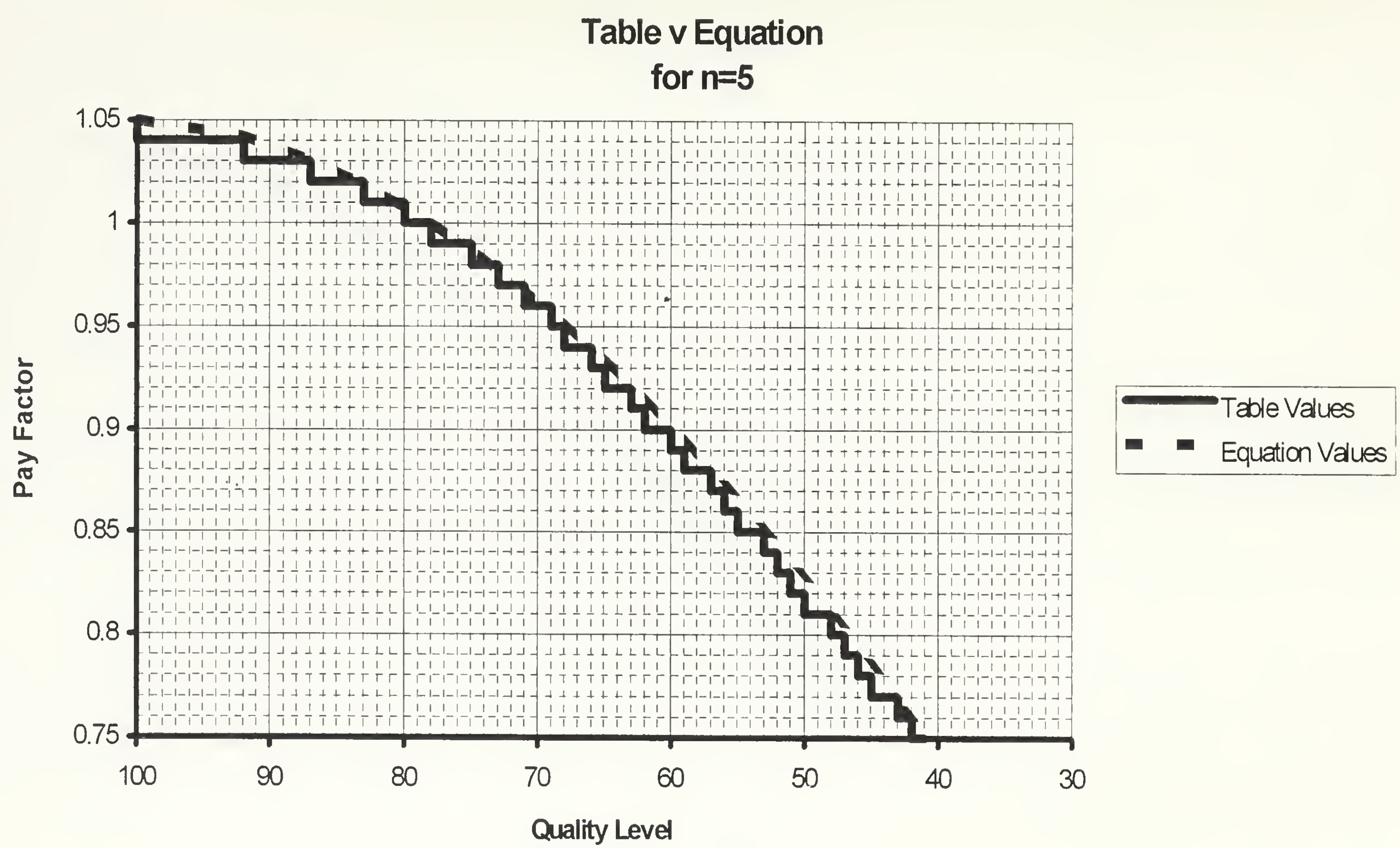


Table v Equation
for n=5
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

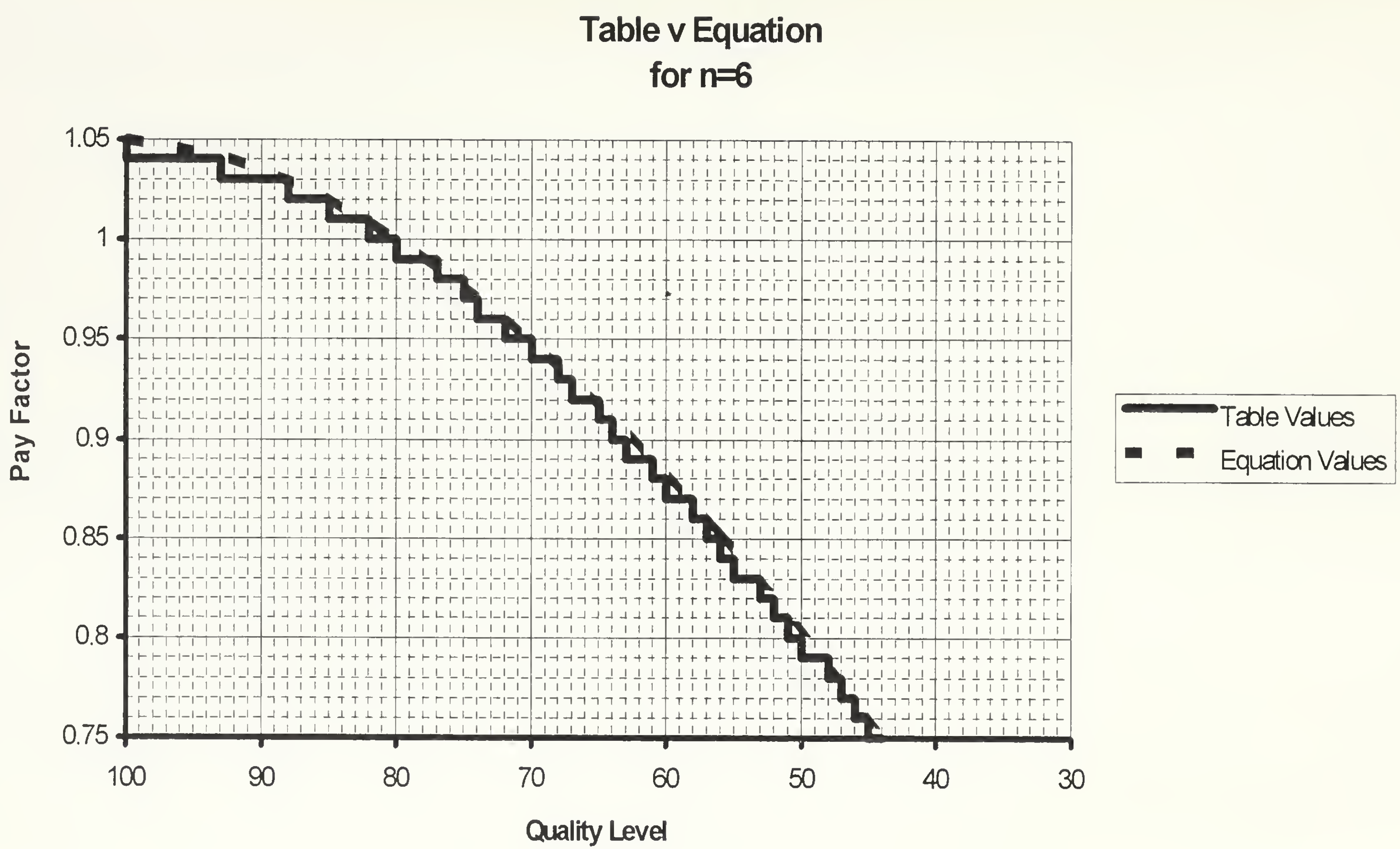


Table v Equation
for n=6
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

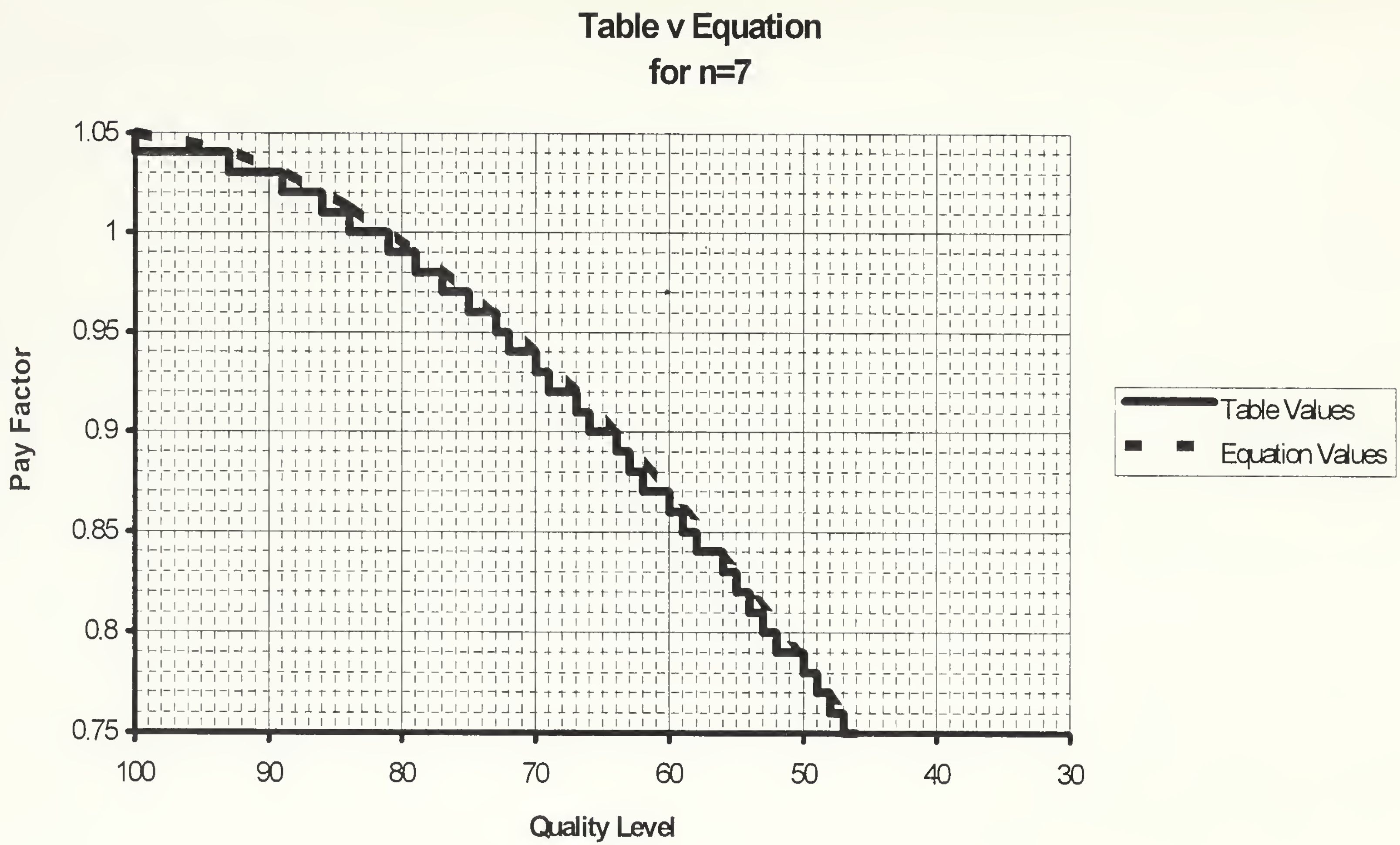


Table v Equation
for n=7
1.05
1
0.95
0.9
0.85
0.8
0.75
Pay Factor
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

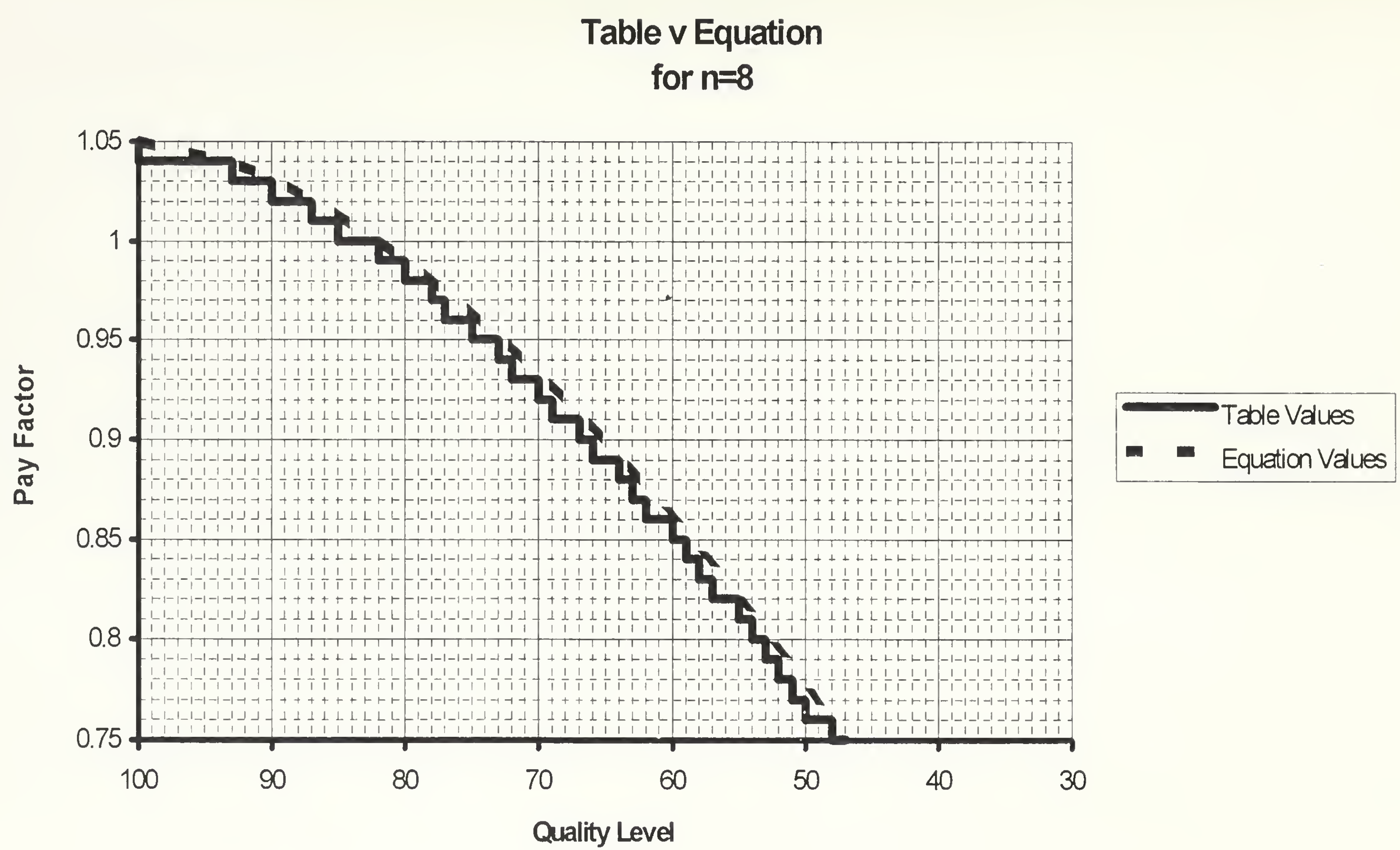


Table v Equation
for n=8
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

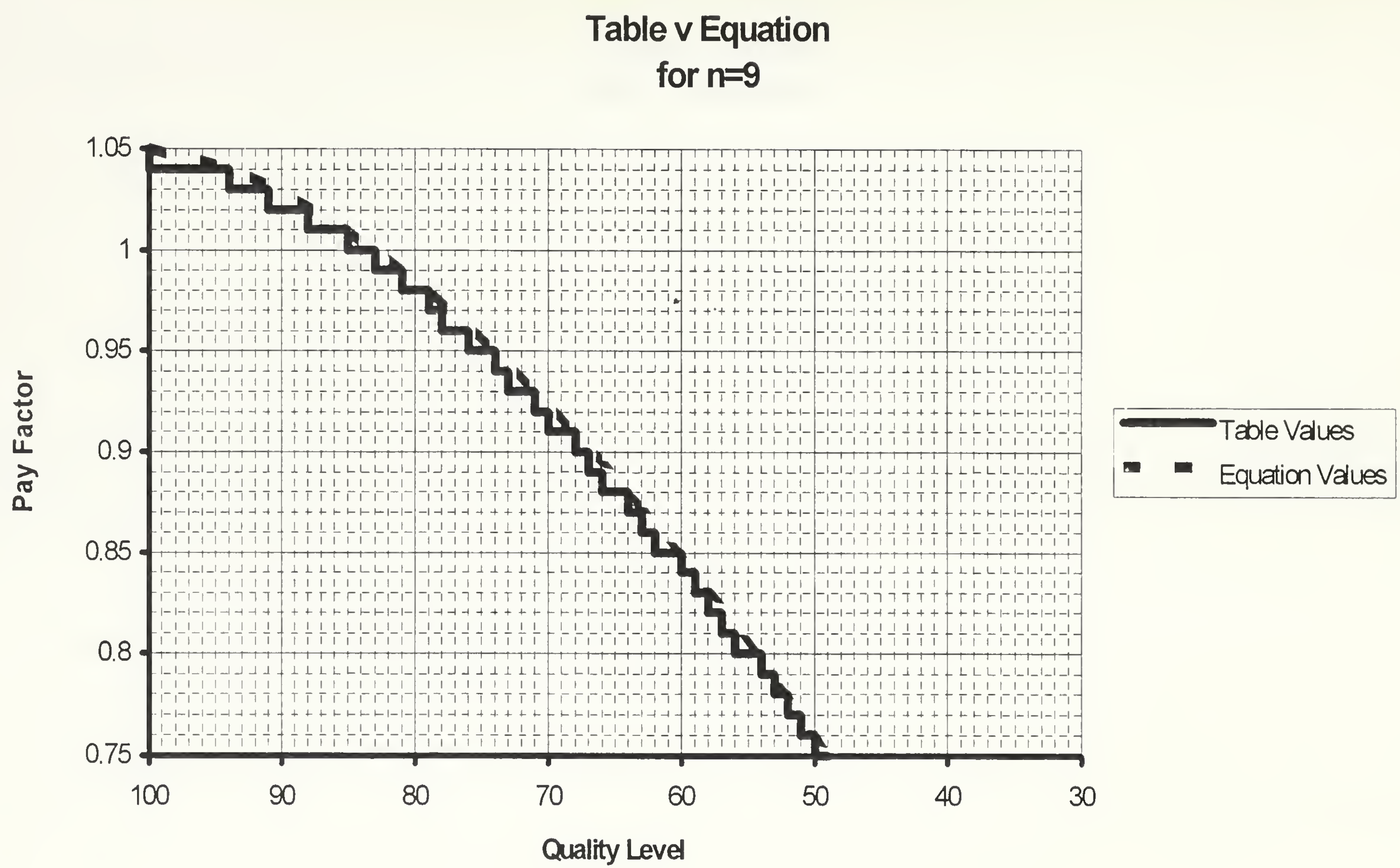


Table v Equation
for n=9
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

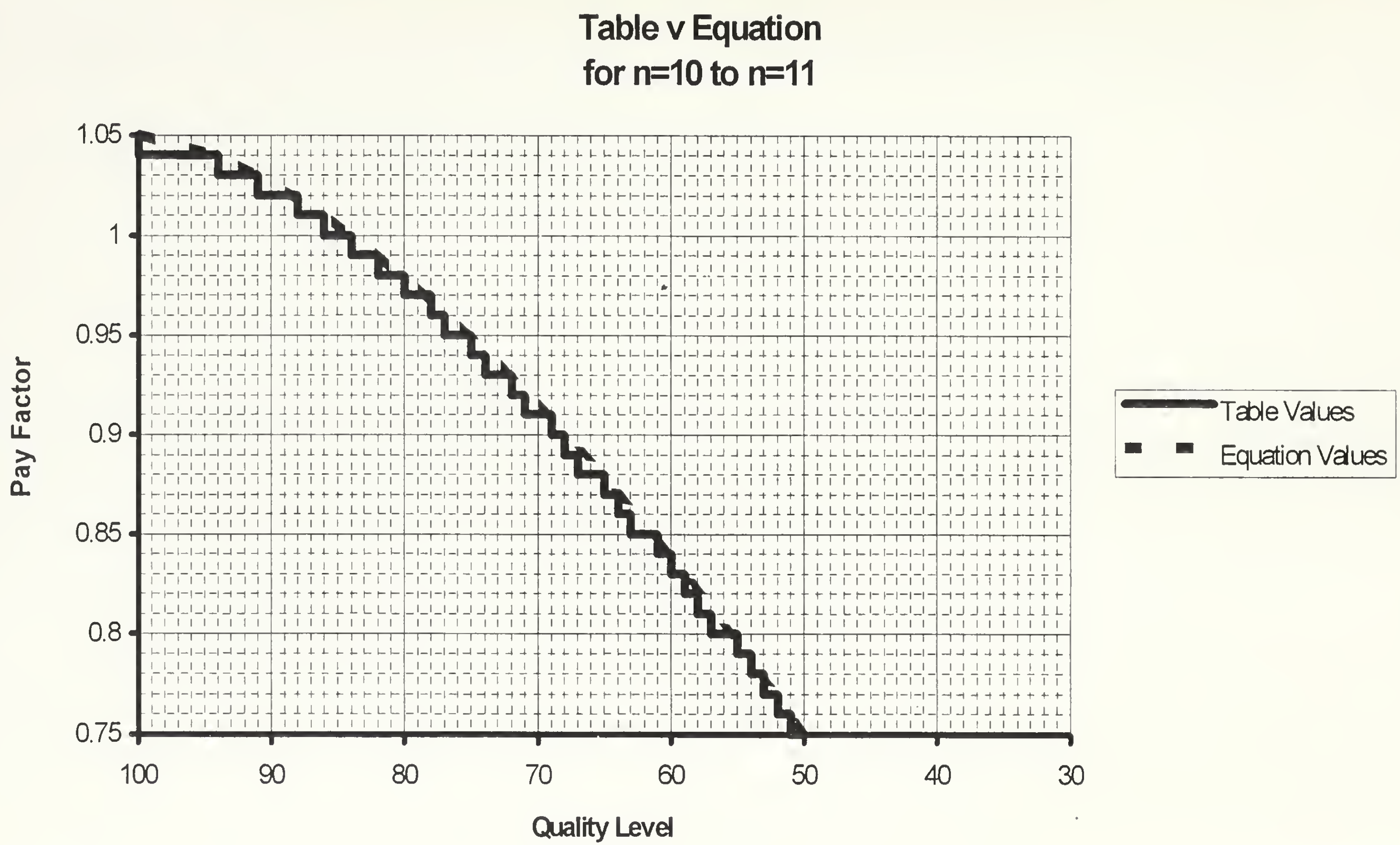


Table v Equation
for n=10 to n=11
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

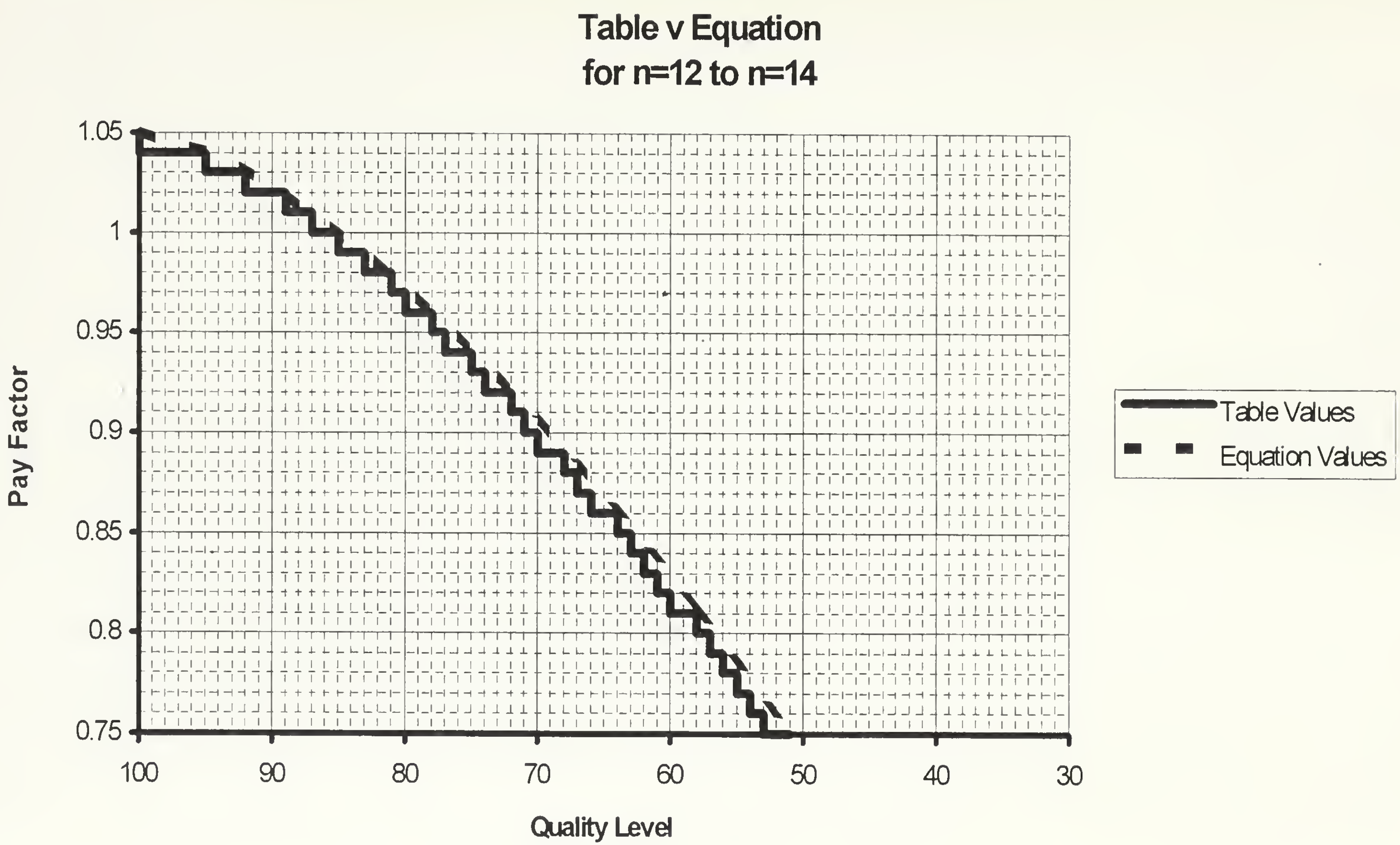


Table v Equation
for n=12 to n=14
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

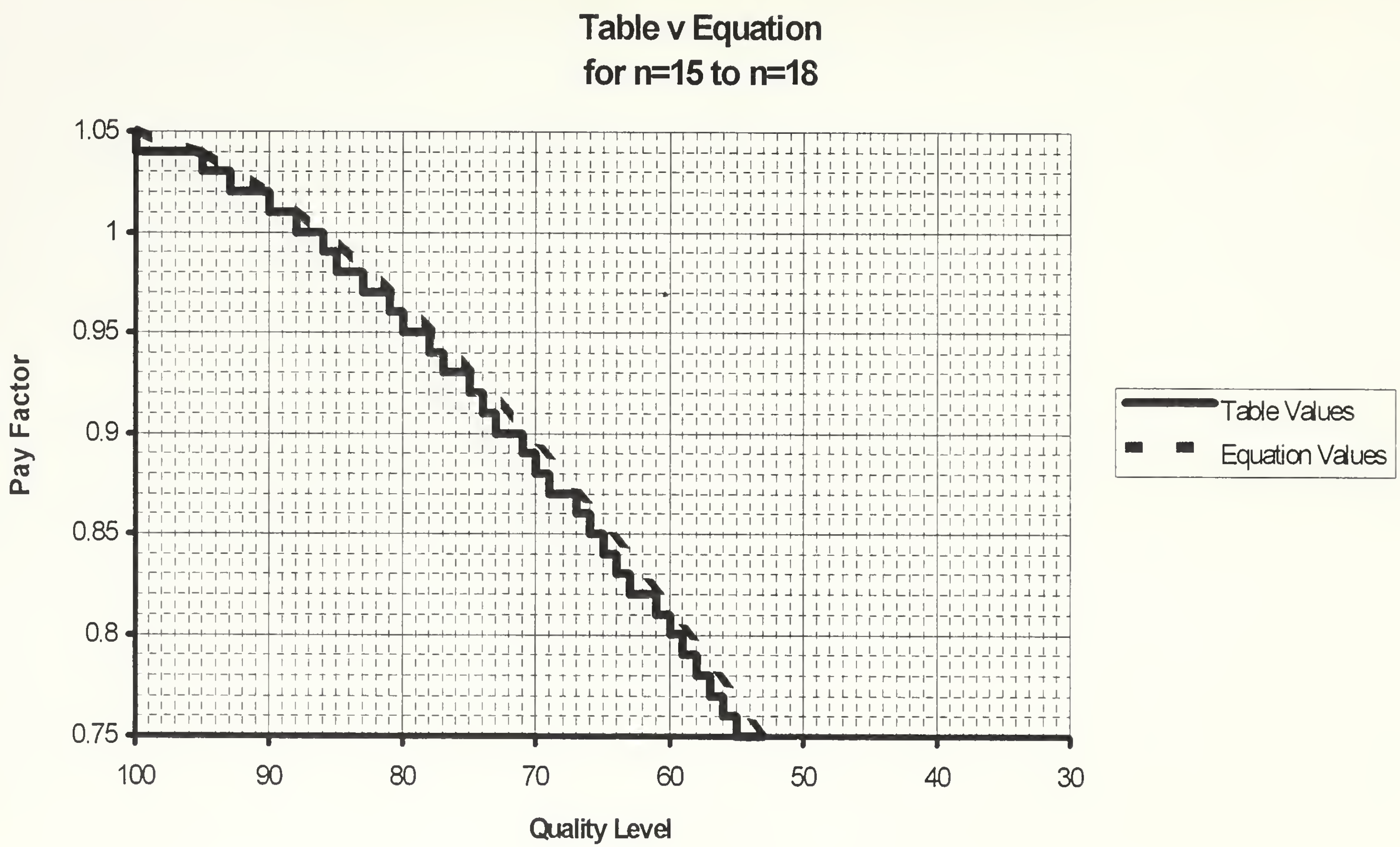


Table v Equation
for n=15 to n=18
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

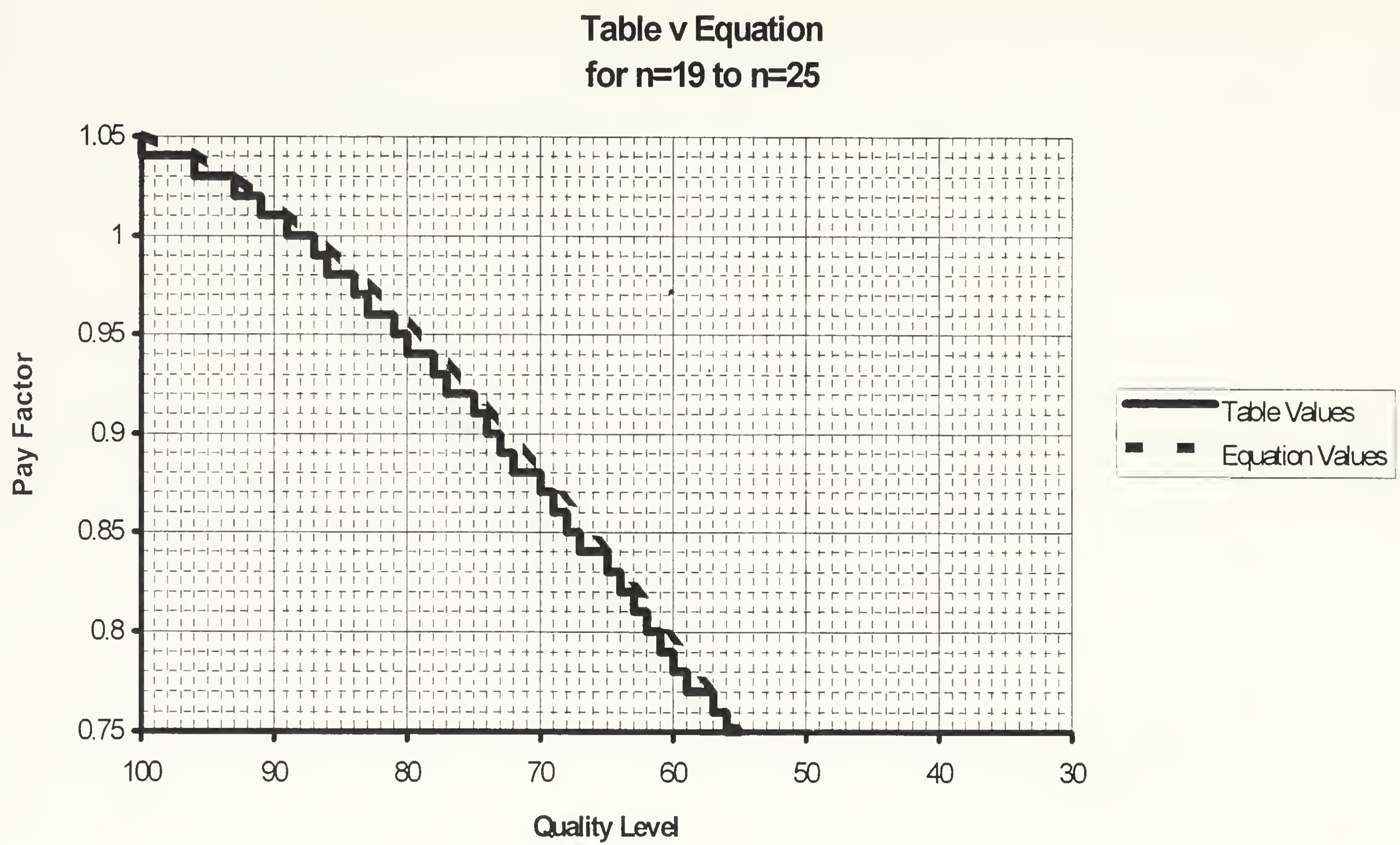
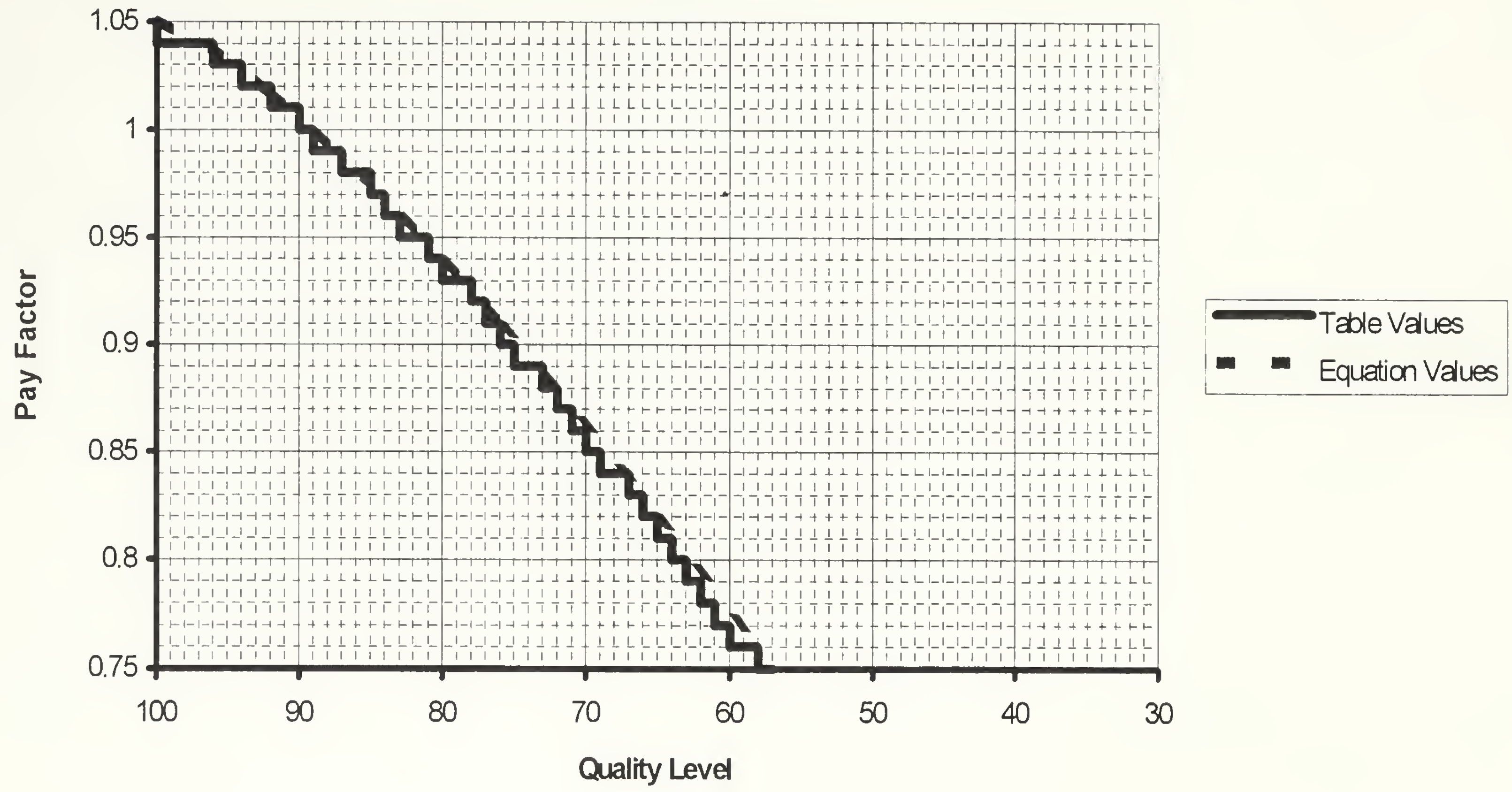


Table v Equation
for n=19 to n=25
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

Table v Equation
for n=26 to n=37



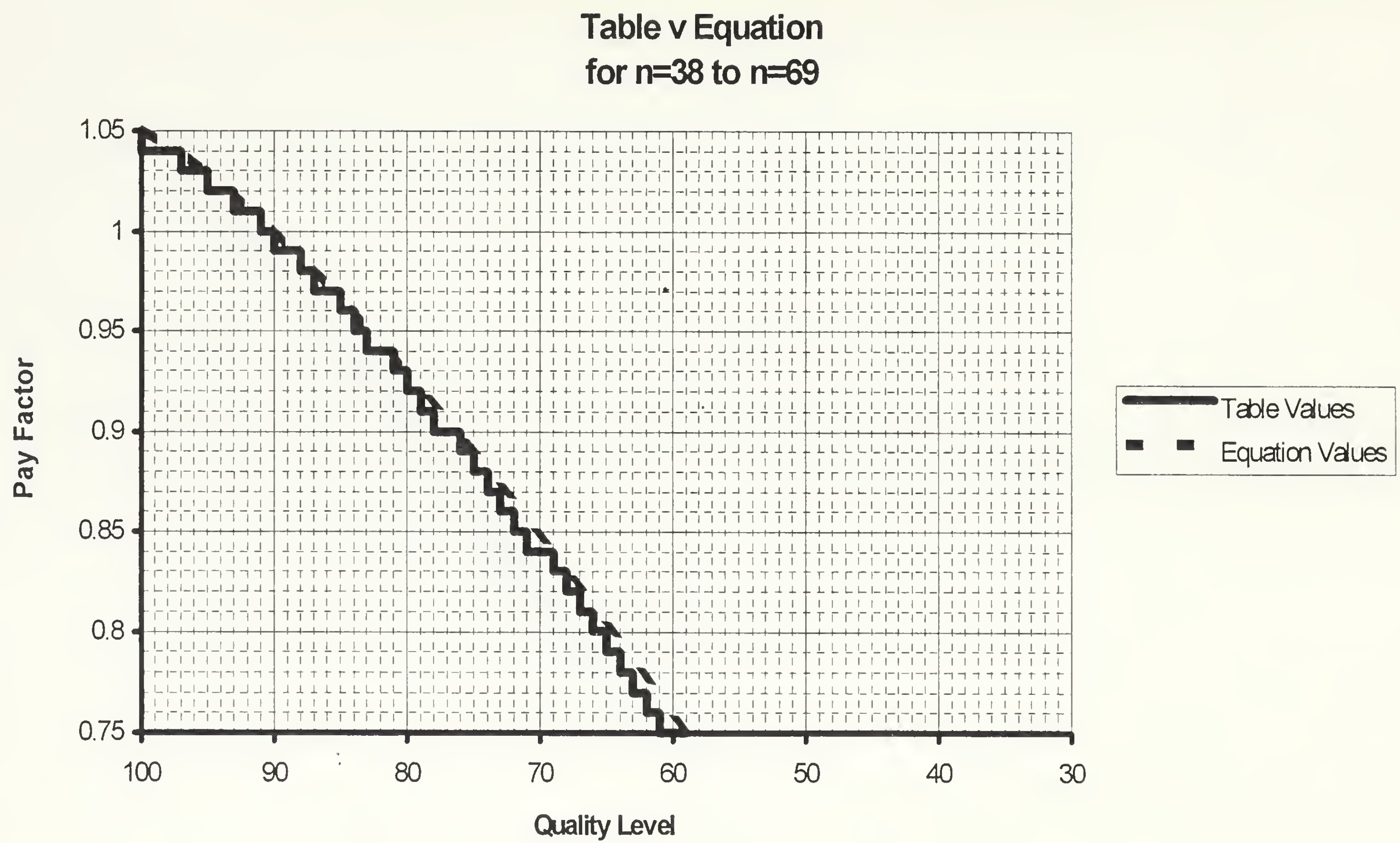


Table v Equation
for n=38 to n=69
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

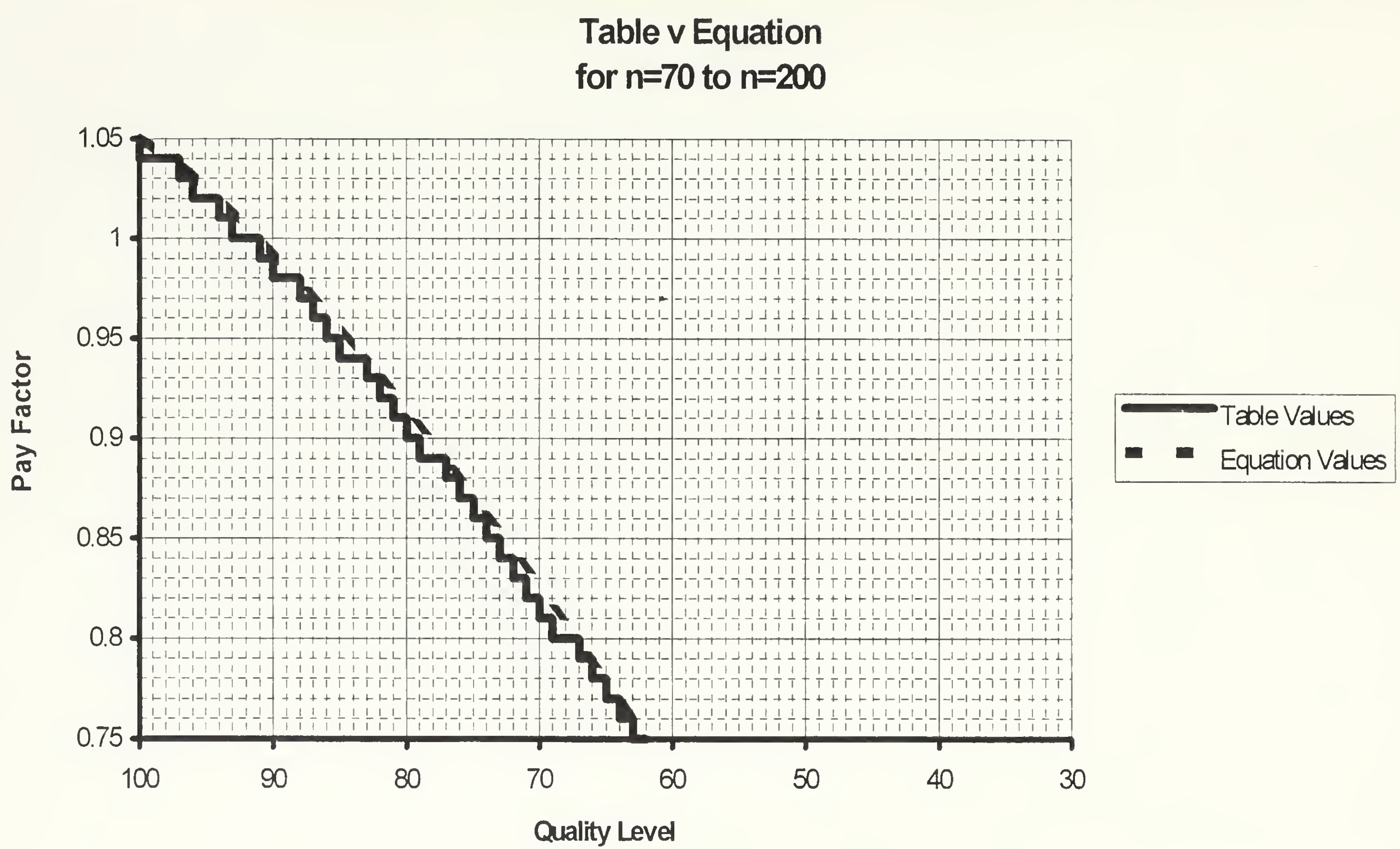


Table v Equation
for n=70 to n=200
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values

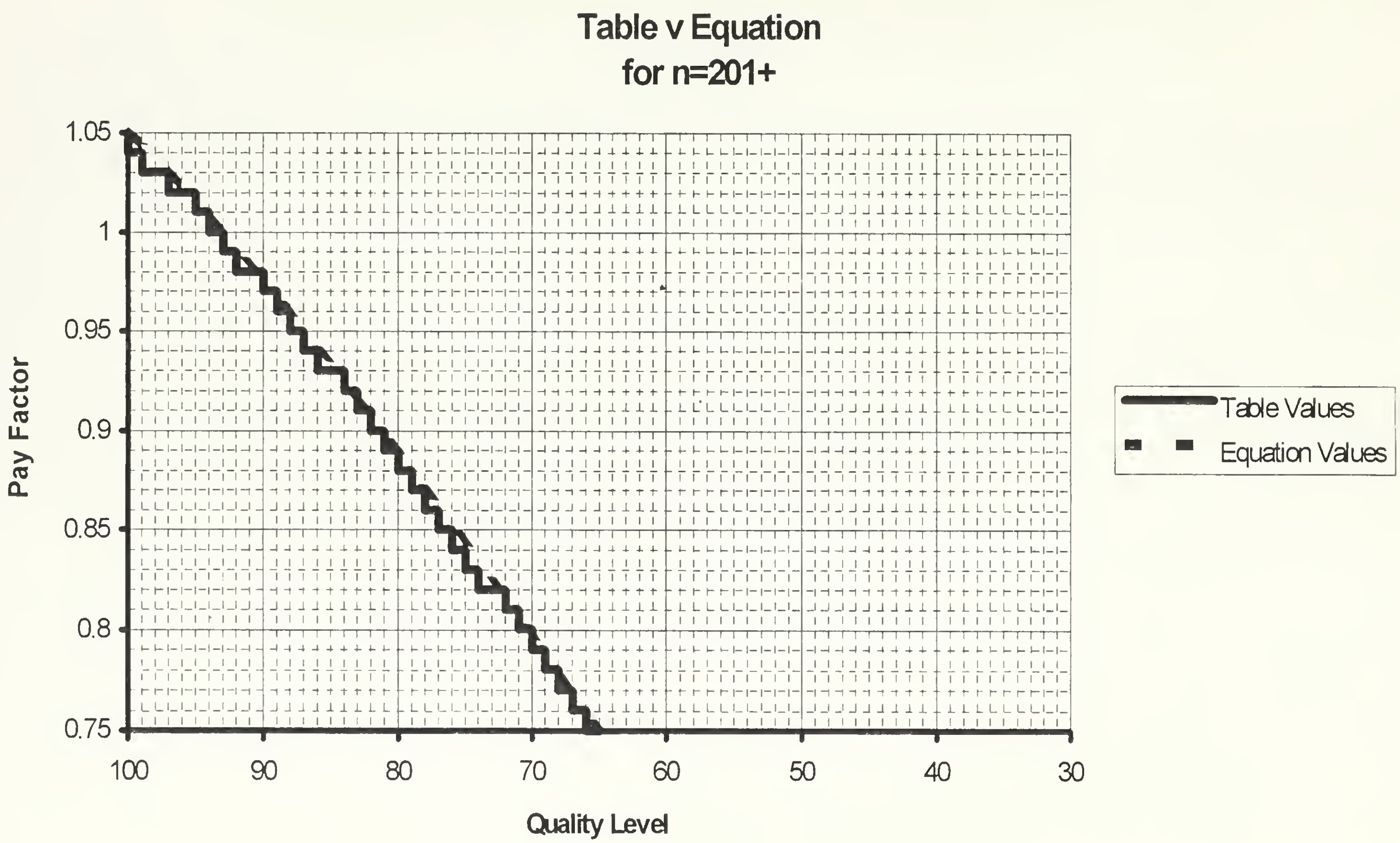


Table v Equation
for n=201+
Pay Factor
1.05
1
0.95
0.9
0.85
0.8
0.75
100
90
80
70
60
50
40
30
Quality Level
Table Values
Equation Values